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XXV. *The Permanent and Temporary Effects on some of the Physical Properties of Iron, produced by raising the Temperature to* 100° *C.* *By* Herbert Tomlinson, *B.A.* *

Introduction.

For many years I have been carrying on researches respecting the effects of stress on the physical properties of matter, and during this period I have become acquainted with certain phenomena, which, though pertaining more or less to most metals, are so conspicuous in iron as to render it worthy of special attention. As these phenomena bear importantly on what Sir William Thomson has designated the thermodynamic qualities of metals, the investigation of which seems to be attracting daily more and more attention, I propose to lay before the Physical Society, from time to time, such information concerning them as a patient study has enabled me to acquire.

On the present occasion I would invite attention to certain remarkable effects produced on some of the physical properties of iron, by merely raising the temperature to a degree not exceeding 100° C.

* Read January 22, 1887.

The Internal Friction of Iron.

If an iron wire be suspended vertically with its upper extremity clamped to a rigid support, and its lower one clamped or soldered to the centre of a horizontal bar of metal, attached to which, at equal distances from the wire, are two cylinders of equal mass and dimensions, and the whole system be set in torsional oscillation, the amplitude of the vibrations will be found to diminish more or less gradually until finally rest ensues. This diminution of amplitude is almost entirely due to two causes, namely, the friction of the air, and the internal friction of the metal. The internal friction of the wire may be measured by the logarithmic decrement of arc for a single vibration, or by $\frac{\log A - \log B}{n}$, where A is the initial arc, and B is the arc after n vibrations. By the aid of Prof. G. G. Stokes's mathematical formulæ *, and an experimental determination by myself † of the coefficient of viscosity of air, I have been able to eliminate the resistance of the air, and to compute the damping effect due to the internal friction of the metal. Where the deformations produced are sufficiently small, I have proved the following laws respecting the logarithmic decrement of arc ‡ :—

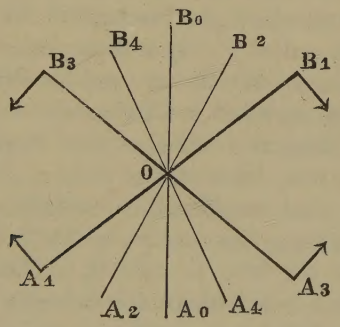
1. It is independent of the amplitude ;
2. It is independent of the vibration-period.

These laws only hold good when the wire has been allowed to rest for a considerable time after any change has been made in the arrangements, and when there has been a large number of oscillations executed previously to the actual testing.

What is the nature of this so-called internal friction of the metal? It cannot resemble fluid friction ; because for such velocities as we have here the friction of fluids is proportional to the velocity. Neither can it resemble altogether the external friction of solids ; because the latter is not nearly so independent of the velocity as is the internal friction, nor would the logarithmic decrement be independent of the am-

* Camb. Phil. Soc. Trans. vol. ix. no. x. (1850).
† Phil. Trans. 1886.
‡ "The Internal Friction of Metals," Phil. Trans. 1886.

plitude. Some experiments by Prof. G. Wiedemann* throw light on the subject. Let $A_0 B_0$ represent the original position of equilibrium of the axis of the bar to which the wire is attached, and let a torsional couple be applied so as to bring the bar to $A_1 B_1$. On reducing the torsional stress gradually to zero the bar will not come back to $A_0 B_0$, but remain in a new position $A_2 B_2$, however small may have been the angle of torsion $A_1 0 A_0$. Again, if the bar be twisted by an equal torsional stress in the opposite direction to $A_3 B_3$ and the stress be then reduced to zero, the bar will remain permanently twisted in the position $A_4 B_4$. Now if we keep on applying and removing the torsional couple in this way, first in one



direction and then in the other, the region $A_2 0 A_4$ will gradually diminish until a minimum is reached. According to Wiedemann, this is exactly what takes place when we allow the wire to vibrate freely; the permanent position of equilibrium is constantly shifted to and fro. Within the regions $A_1 0 A_2$ and $A_3 0 A_4$ the elasticity is perfect, and there is on the whole no gain or loss of energy. The loss of energy experienced in a torsionally vibrating wire arises from the work expended in the region $A_2 0 A_4$ in shifting the permanent position of equilibrium from A_2 to A_4 and back again; and, provided the amplitudes of the oscillations do not exceed a certain limit, the extent of the region $A_2 0 A_4$ is proportional to the amplitude. Wiedemann goes further than this; for he says what is true with respect to the wire as a whole is true

* Wiedemann's *Annalen*, 1879, No. 4, vol. vi.; Phil. Mag. January and February 1880.

with respect to each molecule of the iron, and that the internal friction is really due to the rotation to and fro of the permanent positions of equilibrium of the molecules. We need not stop to discuss here this last point; but what does seem probable is, that the main part of the loss of energy is experienced as the bar swings from A_2 to A_4. When, as in my own experiments, the deformations produced by the oscillations are very small, it would seem that the positions A_2, A_4 are really subpermanent rather than permanent; and if time were given and the molecules agitated, the bar would of itself return to the position A_0, when the torsional couple was reduced to zero.

I have said, that as the wire oscillates, the region $A_2\,0\,A_4$ becomes narrower and narrower, and Wiedemann speaks of the period during which the diminution takes place, as "the accommodation period." My own experiments have verified the results of those of Wiedemann and Sir William Thomson *, in showing that repeated oscillation will reduce the internal friction; but they also show a very large influence to be exerted by long rest, either with or without oscillation, and have further proved that considerable diminution, both temporary and permanent, can be produced by merely raising the temperature of the wire to 100° C. Thus a well annealed iron wire, when tested about ten minutes after suspension, was found to have a logarithmic decrement due to the internal friction of the metal of ·003011, after one hour of ·001195, and after one day of ·001078. After the last period the friction became sensibly constant, and after four days was found to be still the same; the wire had apparently "accommodated" itself as far as possible.

Great, however, as was the reduction of the internal friction produced by oscillation and rest, the minimum had by no means been reached; for on repeatedly heating the wire to 100° C. and then allowing it to cool, the logarithmic decrement rapidly diminished, until after six days, on each of which the wire was heated to 100°C., and then allowed to cool slowly, it became only ·000412, when further repetition of the above process ceased to sensibly affect the friction. The greater part of the diminution occurred after the first heating and cooling, but several repetitions were necessary to produce the minimum mentioned above.

* Proc. Roy. Soc. May 18, 1865.

Still more marvellous is the temporary effect of a rise of temperature not exceeding 100° C. on the internal friction of annealed iron. A careful examination of the above specimen at temperatures ranging between 0° C. and 100° C. revealed the astonishing fact that, at a temperature of 98° C., the logarithmic decrement was only ·000112, and was considerably less than one fourth of its amount at 0° C.* At 98° C. the friction was a minimum, further rise of temperature resulting in increase of the logarithmic decrement. It may perhaps assist us in forming some notion of the very small amount of internal friction in the above specimen of iron at 98° C., if we estimate the number of vibrations which would be required before the amplitude would be reduced to one half of its initial value by molecular friction only: this number is nearly 3000. It follows that if we could make the wire and its appendages vibrate *in vacuo*, and maintain the temperature constantly at 98° C., with a vibration-period of ten seconds, more than eight hours would elapse before an initial amplitude of 100 would be reduced to 50.

The internal friction of the wire when reduced to its minimum by all the above-mentioned processes was only *one thirtieth* of its original amount.

When the wire has fully "accommodated" itself, a very small cause will disturb the accommodation: a mechanical shock, a change of load, a slight rise or fall of temperature, or even rotation of the molecules by magnetic stress, will necessitate fresh oscillations before the friction again reaches its minimum. Consequently, if a wire be raised to 100° C., and be then cooled again rather quickly, it does not immediately regain the accommodation which it had before heating. The time taken by the wire to reaccommodate itself when the accommodation has been disturbed by change of temperature, depends considerably upon the direction of the change. Thus, when the wire is raised from the temperature of the room to 100° C., the reaccommodation is effected in a very much shorter time than when the accommodation has been disturbed by lowering the temperature from 100° C. to the temperature of the room.

* The temporary diminution of the internal friction of annealed iron was shown at the meeting of the Society.

I am inclined to regard both the temporary and permanent alterations of the internal friction of iron, which are produced by rise of temperature not exceeding 100° C., to be partly due to mere agitation of the molecules, but the permanent effects do not seem to be entirely due to molecular agitation ; for the maintaining of the temperature at 100° C. for some time does not bring down the friction anything like so much as repeated heating and cooling. It would seem that the slow shifting backwards and forwards of the molecules induced by the last process is in a great measure to be credited with the permanent diminution of friction, in the same manner that the shifting backwards and forwards of the molecules caused by torsional oscillation has been shown to produce permanent diminution.

The Longitudinal and Torsional Elasticity of Iron.

It might readily be imagined that since the internal friction of iron is so considerably altered by change of temperature, the elasticity would be correspondingly affected. This, however, is not so ; both the torsional and longitudinal elasticity of iron are affected by raising the temperature to 100° C., but not nearly to the same extent as the internal friction. Thus an annealed iron wire, when suspended ready for torsional vibration, was heated slightly by passing the flame of a Bunsen's burner rather quickly up and down it several times. The time of vibration before heating was 1·154 second ; and in 5 minutes, 35 minutes, and 245 minutes after cooling was 1·147, 1·142, and 1·136 second respectively. Here we have a small, though distinct, permanent increase of elasticity ; and it will be noticed that time is an important element in the amount of increase. Again, by a very carefully conducted set of observations, I have shown that the torsional elasticity of annealed iron is temporarily *decreased* to the extent of 2·693 per cent., when the temperature is raised from 0° to 100° C.*

Again, an annealed pianoforte-steel wire, when tested with a certain load at the temperature of 12° C., was temporarily elongated to the extent of 1·502 half-millimetres; when heated to 100° C. in an air-chamber the elongation was 1·487 half-millimetres; and when cooled again and tested 24 hours afterwards,

* Proc. R. S. No. 244, 1886.

was elongated by 1·450 half-millimetres *. Thus, as with the torsional elasticity, there was a permanent increase of elasticity, and a *temporary decrease* of 2·58 per cent. Time in this case also is an important element, for the elasticity immediately after cooling was very appreciably less than when a long rest had been given.

Similarly, I have shown both for longitudinal and torsional elasticity that an iron wire, after having been permanently extended by traction, has its elasticity very perceptibly increased by long rest. It is also well known that the portative power of a magnet can be considerably increased by putting on the load by small quantities at a time, with long intervals of rest between.

These and other considerations prove beyond a doubt, that if the molecular arrangement of iron be disturbed by any kind of stress whatever, exceeding a certain small limit, the molecules will not assume at ordinary temperatures those positions which will secure a maximum of elasticity, until a rest of many hours has been given.

The Velocity of Sound in Iron.

According to Wertheim, the velocity of sound in iron and steel is increased by a rise of temperature not extending beyond 100° C.† Now in no sense whatever is this statement correct. It is true that the longitudinal elasticity of iron, as determined by the method of statical extension, may be found greater at 100° C. than at 0° C., provided we begin with the lower temperature first and the wire has not been previously tested at 100° C. But, as we have seen, the apparent *temporary* increase of elasticity is really a *permanent* one; and if the wire be repeatedly heated to 100° C. and then cooled, subsequent tests will always show a less elasticity at the higher temperature than at the lower, *if sufficient rest after cooling be allowed.* When, however, we come to such small molecular displacements as are involved in the passage of sound through a wire, even the apparent increase of elasticity, mentioned above as taking place at the first heating, vanishes. I have been able to prove that, when an iron or steel wire is thrown

* Phil. Trans. Part I., 1883, p. 130.

† *Ann. de Chimie et de Phys.* 3me série, 1844, p. 421.

into longitudinal vibrations, so as to produce a musical note, the pitch of this note becomes *lower* as we *raise* the temperature even when the wire is heated for the first time*.

It seems rather strange that the error should have been so long allowed to remain uncorrected; for it has been known for many years that the pitch of a steel tuning-fork is lowered by small rises of temperature to a greater extent than would follow from mere change of dimensions. Calling the frequency of the fork n, we have†

$$n = \frac{\kappa b}{2\pi l^2} m^2, \quad \ldots\ldots\ (1)$$

where m is an abstract number,

b is the velocity of sound in steel,

l is the length,

κ the radius of gyration of the section about an axis perpendicular to the plane of bending.

If D be the thickness of the fork,

$$\kappa = \frac{D}{\sqrt{12}}.$$

We may therefore obtain

$$b = \frac{4\pi\sqrt{3}\, l^2}{m^2 D} n. \quad \ldots\ldots\ (2)$$

From (2) and the value of the coefficient of thermal expansion of steel, it follows that if the pitch of the note is lowered by rise of temperature to the extent to which it is known to be, the velocity of sound *must* be lowered also. Indeed from the coefficient of thermal expansion of steel and from my own determination of the effect of change of temperature on the longitudinal elasticity of steel, I have calculated what would be the lowering of pitch of a certain fork, and find it in sufficient accord with the lowering of pitch as determined by direct experiment.

Wertheim inferred the increase of velocity of sound in iron and steel from the apparent increase of longitudinal elasticity produced by rise of temperature. From his experiments on

* The *lowering* of pitch produced by *rise* of temperature was shown before the Society.

† Lord Rayleigh's 'Theory of Sound,' vol. i. p. 219.

the longitudinal elasticity of these metals I have collected the following :—

Metal.	*Increase* per cent. of elasticity between 15° to 20° and 100° C.
Annealed iron	4·90
Annealed iron wire	6·91
Annealed cast steel	2·79
Annealed steel wire	23·20
Steel tempered blue	5·18

whereas my own experiments show that both the torsional and longitudinal elasticities of iron and steel are *decreased* by about $2\frac{1}{2}$ per cent. when the temperature is raised from 0° C. to 100° C.

I have dwelt longer than I should otherwise have done on this part of my paper because I find that even our best text-books relating to Elasticity and Sound still retain what I am convinced is an error.

XXVI. *Measuring-Instruments used in Mechanical Testing.* *By* Prof. W. C. Unwin, *F.R.S.**

The determination of the exact distance between two fine marks on a standard of length is an operation of some difficulty, as is well known to physicists. But that operation is free from many of the difficulties which attend the measurements which have to be made in the engineering laboratory. Among these the determination of the modulus of elasticity (Young's modulus) of a bar by measuring its change of length by *stress* is one of the most important. Now the bars subjected to test are usually, in the part which can be measured, not more than 10 inches in length ; and the whole elastic extension of such a bar is generally only about 0·007 inch. It is obvious, therefore, that measurements must be made with considerable accuracy and refinement to be of any value. But the bar cannot be placed in a position convenient for measurement; and the attachments to the testing-machine are more or less in the way of the measuring-apparatus to be applied. The bar itself is a somewhat rough bar, the form of

* Read January 22, 1887.

which must not be interfered with to facilitate the measurements. Then also bars of very different forms have to be tested, flat and round, of various widths and diameters; and the measuring-apparatus must be applicable to all these with equal readiness. Last, but not least, the work of an engineering laboratory is pressing, and measurements must be carried out with rapidity.

In some cases, two diamond scratches have been made on the bar, and the distance between these measured by two micrometer-microscopes. Apparatus of this kind is awkward to apply on the testing-machine, and tedious to adjust and read.

A cathetometer has been used. But then two adjustments have to be made, and two readings taken for each elongation. Also the limit of accuracy of the cathetometer is hardly sufficient for the purpose.

Very often mechanical magnification by a lever is adopted. But there are some difficulties in satisfactorily attaching a lever-apparatus to the bar: if a leverage of 100 to 1 is adopted, the fulcrum distance becomes very short, and the range of the apparatus is limited. There is also some difficulty in the calibration of the instrument to determine the value of the readings.

A micrometer-screw is sometimes used as a means of mechanical magnification. With this there is, again, the difficulty of suitable attachment to the bar; and, as generally used, it is difficult to ascertain when exact contact of the screw is obtained without excessive pressure.

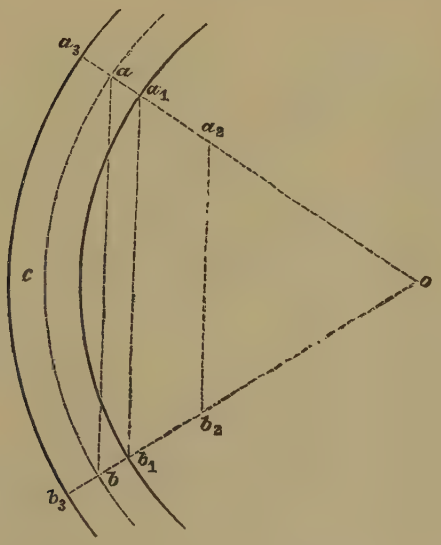
There is a special difficulty in measuring the elongation of ordinary test-bars, which has been overlooked in the construction of most of the apparatus of this kind. It is difficult to get test-bars which are rigidly straight. Even if the test-bar is strictly straight, it is difficult to hold it in the testing-machine, so that the resultant of the stress on any cross section passes strictly through the centre of figure of the section. Now if this condition is not satisfied, the bar becomes curved during the test. The straightening of an initially curved bar, or the curving of an initially straight one, introduce errors in the measurements of very considerable amount.

If the measurements could be made at the axis of the bar,

the errors of this kind with any amount of curvature likely to occur would not be very serious; but this is of course impossible. The best that can be done is to measure at the surface of the test-bar. But, in straightening, the surface of the bar on one side lengthens and on the other shortens, and thus introduces a not inconsiderable error of measurement. If, as in many forms of elongation measuring-apparatus, the measuring-points are two inches or more from the axis of the bar, the errors become very large relatively to the elongations to be measured.

Let fig. 1 represent a bar bent in the plane of the paper,

Fig. 1.



the centre of curvature being O. Then, if measurements could be made on the axis of the bar, between the points $a\,b$, the straightening of the bar would introduce an error equal to the difference of the length of the chord $a\,b$ and arc $a\,c\,b$. With any amount of curvature likely to occur in a test-bar, this error would not be very serious. Generally, however, the best that can be done is to measure the distance between points $a_1\,b_1$ on the surface of the bar. Then, since by straightening the lines $a\,O$, $b\,O$ become parallel, the error

introduced is the difference between $a_1 b_1$ and the arc $a c b$ and this is much more serious. Most commonly, however measurements are made between points on clips fastened t the bar at 1 or 2 inches distance from its surface, such a $a_2 b_2$. Then the error introduced by straightening is th difference between a_2, b_2 and the arc $a c b$; and this may be serious error, even with a very small amount of initial cur vature.

If simultaneous measurements are taken of $a_1 b_1$ and $a_3 b_3$ the mean of these will have no greater error than the mea surement of $a b$. That is, the mean of measurements on tw sides of the bar reduces the error due to initial or induce curvature to the same amount as a measurement actuall made at the axis of the bar.

Prof. Bauschinger, of Munich, appears to have been th first to recognize the importance of this double measuremen symmetrically on the two sides of the bar. He has alway used an apparatus in which a finger, or touch-piece, attache to one end of the bar, presses on a roller attached to the othe end. As the bar extends, the roller rotates by friction agains the finger. A mirror is attached to the roller; and the amoun of rotation is observed by noting the image of a scale in th mirror through a reading-telescope. In this way measure ments to $\frac{1}{125000}$th of an inch can be taken. To eliminat errors due to curvature, two rollers are placed, one on eac side of the bar, and two sets of readings are taken. Thi involves the adjustment of two instruments and the taking o two sets of readings. But the principle is perfect; and n more accurate measurements than Bauschinger's have pro bably been made.

Touch-Micrometer Extensometer.—The first instrument use by the author was a kind of callipers. Two bars, one sliding in the other, could be set by touch to the distance betwee two fixed clips on the test-bar. A scale was engraved o silver on one bar; and the distance of the nearest division fro a fixed zero-mark on the other was taken by a microscope micrometer. Readings could be taken to $\frac{1}{10000}$th of an inch The instrument is easy and rapid to use. Readings can b taken on both sides of the test-bar; and the readings are direc

on to a carefully graduated scale, so that no calibration of the instrument is necessary.

Screw-Micrometer Extensometer.—This aims at obtaining the extension along the axis of the bar by a single reading. Two clips are fixed on the bar, each by a pair of steel points, one on each side, gripping the bar in a plane through its axis. If, then, these clips can be made to preserve the same relative position to the bar, the middle points of the clips will move in the same way as points on the axis of the bar. Fig. 2 is a diagrammatic sketch of the apparatus. *a a* and *b b* are the clips on the test-bar, fixed to it by points in its middle plane.

Fig. 2.

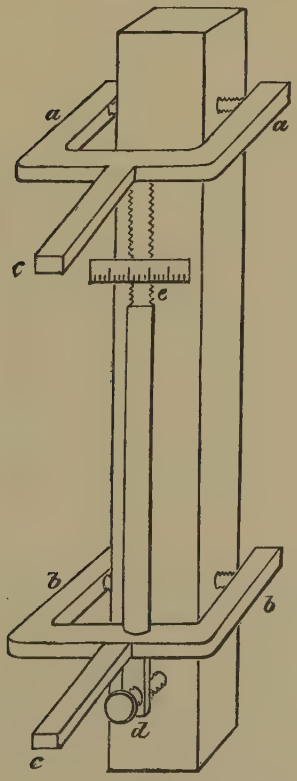


Fig. 3.

c c are projections on the clips, to which are fixed delicate spirit-levels; *d* is a small screw which just touches the test-bar;

e is a micrometer-screw with graduated head, which supports the upper clip on the lower clip. In use the lower clip is first levelled by the screw *d*; then the upper clip is levelled by the micrometer-screw, and a reading taken. The clips being always accurately levelled, in a plane perpendicular to that in which the four points attaching the clips to the test-bar lie, the micrometer-readings are the distances between the middle points of the two parallel clips; and their differences are the mean of the elongations on the two sides of the test-bar, or virtually are readings at the axis of the test-bar. Readings to $\frac{1}{10000}$th of an inch can be taken.

Roller-and-Mirror Extensometer.—Fig 3 is a diagrammatic sketch of another instrument on the same principle. *a* and *b* are two clips similar to those in fig. 2; the lower clip is supported on the test-bar by a screw *d*; the upper clip is supported on the lower by a stay-bar with knife-edges, *e*. At *r* and *m* are the roller and mirror, the axis of these being at the same distance from the knife-edge of the stay-bar as the set screw of the clip. A touch-piece or finger, *f*, attached to the lower clip presses on the roller. If the bar extends, the roller approaches the lower clip by an equal amount; it turns against the finger *f*; and the amount of rotation is read by a telescope and scale. This instrument will easily read to $\frac{1}{100000}$th of an inch. The roller being at the centre of the clip, its movement is the mean of the elongations on the two sides of the test-bar.

The author showed a third instrument on the same principle, for obtaining the compression of small blocks of stone.

XXVII. *On an Arc-Lamp suitable to be used with the Duboscq Lantern.* *By* Professor Silvanus P. Thompson, *D.Sc.**

[Plate III.]

The lamp devised by Foucault and Duboscq, and supplied for so many years by the famous house of Duboscq, fails to fulfil the electrical requirements of the modern physical laboratory, though it has rendered excellent service in the past. Yet the lantern and optical adjuncts of the standard

* Read March 27, 1886.

pattern of Duboscq are so widely used that it seemed desirable to find some other arc lamp which, while fulfilling the electrical requirements of the case, could be used with the Duboscq lantern.

Before describing the lamp which I have for twelve months employed for this purpose, I propose to state the conditions to be fulfilled, and the reasons why the old Duboscq lamp fails to fulfil them.

The modern physical laboratory is usually supplied with electric energy under one of two alternative conditions, namely either at constant potential or with constant current; more usually under the former condition. If supplied from a dynamo the dynamo may be either series-wound, shunt-wound, or compound-wound. If supplied from accumulators the accumulators will work at constant potential, and will have a very small internal resistance.

The arc-lamp for laboratory use must be capable of working under the given conditions. No doubt the Duboscq lamp worked fairly when supplied with current from 50 Grove's cells. But in a laboratory where there is another and better and less wasteful source of supply, 50 Grove's cells are not desirable. Though 40 accumulators have an electromotive force almost exactly equal to that of 50 Grove's cells, the Duboscq lamp does not work well with them unless a resistance of several ohms is intercalated in the circuit to represent the internal resistance of the Grove cells; and even then the Duboscq lamp does not, for certain reasons, work as satisfactorily as the lamp to be described, and its cost is about three times as great.

In every arc-lamp for optical purposes there must be mechanism adapted to perform the four following actions:—

1. To bring the carbons together into initial contact.

2. To part the carbons suddenly, and with certainty, to a short distance—about 3 millimetres—apart. This action is technically called "striking" the arc.

3. To supply carbon as fast as it is consumed, by moving one (or both) of the pencils forward into the arc. This action is called "feeding" the arc.

4. To so move the carbons, or their holders, that the luminous points retain the same position in space at the proper

focus of the optical system. This action is called "focusing" the arc.

It may be remarked, in passing, that the feeding mechanism of many lamps also performs the action, set down as No. 1 of the above, of bringing the carbons into initial contact preparatory to striking the arc.

In many arc-lamps the attempt is made to unite the striking and feeding mechanisms in one; but in many lamps, and in the one I have to describe, the striking and feeding mechanisms are distinct. The striking mechanism in all the arc-lamps of commerce consists of an electromagnet or solenoid arranged in the main circuit of the lamp, the armature or plunger of the same being mechanically connected with one or both of the carbons, so that when, by the turning on of the current through the touching carbons, there is a great rush of current, the attraction of the electromagnet or solenoid shall instantly part the carbons and strike the arc. In the majority of the commercial arc-lamps it is the upper carbon only that is raised to strike the arc; in a few other lamps, and in the one I am using, the lower carbon is depressed. In one of the older patterns of the Duboscq lamp the lower carbon was also thus directly acted upon, its holder being attached to the armature of an electromagnet beneath it. The same is true of the Serrin lamp. But in the Duboscq-Foucault lamp the arc is struck in a different way. The two carbon-holders are connected by racks to a clockwork gearing which either parts them or brings them together, the movement being driven by a double train of wheels, either of which can be released in turn. The weight of the upper carbon-holder drives the train that moves the carbons together; a coiled spring drives the train that parts the carbons. Whether either of the trains, or neither of them, shall be released is determined by the position of a double-toothed detent which, placed between the final spur-wheels of the two trains, locks both of them when in its mean position, but releases one or other when shifted to right or left. The position of this detent is determined by the current through the lamp, it being attached to one end of a three-arm lever, the two other ends of which are respectively attached to the armature of the controlling electromagnet and to an opposing spiral spring.

When the moment of pull of the electromagnet upon its armature is greater than that of the opposing spring, the detent is pulled over one way, releasing the approximating train of wheels while retaining locked the parting train. When the moment of the pull of the opposing spring exceeds that of the electromagnet on its armature, the detent is pulled over the other way, locking the approximating train and releasing the parting train. When the pull of the electromagnet exactly balances that of the opposing spring, both trains are locked. Now when the current is at first turned on, there is a sudden pull upon the armature of the electromagnet; but the carbons are not instantly parted, partly because of the inertia of the train of wheels, and partly because of the backlash of the mechanism. Two or three seconds may elapse before the arc is struck. This delay is serious, either when working with dynamo or with accumulators. If the dynamo is shunt-wound, the shortcircuiting even for this short period demagnetizes the field-magnets. If the dynamo is series-wound or compound-wound, or if accumulators are being used, there is overheating during the period of delay. Supposing, however, the arc to be struck, then the inertia of the train of wheels makes itself evident in another way; for it parts the carbons too far, producing a long arc of considerable resistance; and as the current then drops below its normal value, the armature goes over the other way, and the other train of wheels is momentarily released. This alternation between the two trains, which often lasts for some time, produces a disagreeable instability.

The feeding mechanism of arc-lamps next comes in for consideration. The object of the feeding mechanism is to supply carbon as fast as it is consumed, and so keep the light constant. But the light cannot be kept constant unless the consumption of electric energy in the arc is constant. The electric energy is the product of two factors—the current through the arc, and the difference of potential between the electrodes. Calling the current i and the potential difference e, it is the product ei which is to be kept constant. Now, as remarked at the outset, the very conditions of modern electric supply are that either e or i is maintained constant, the usual arrangement in commercial lighting being i constant for arc-lamps in series,

and e constant for glow-lamps in parallel. One of the two factors being a constant by the conditions of the supply, the other factor must be kept constant by the feeding mechanism. Or, in other words, the variations of the other factor should be made to control the action of the feeding mechanism. The mechanical part of the feed may consist of a train of wheels driven by the weight of the carbon-holder or by a spring, or it may consist of a friction-clutch holding the carbon from sliding forward, or of a worm-gearing or any other; but it must be controlled by an electric mechanism of one of the two following kinds. For keeping i constant, the feeding mechanism must be controlled by an electromagnet (or solenoid) placed in the main circuit, working against an opposing spring or weight. For keeping e constant, the feeding mechanism must be controlled by an electromagnet (or solenoid) placed as a shunt to the arc, and working against an opposing spring or weight. In the latter case, if for any reason the arc grows too long, the potential at the terminals will rise, more current will flow around the shunt, which will then overcome its opposing spring (or weight), and will release the feeding machinery until balance is restored. The use of the shunt, introduced first by Lontin, enables arc-lamps to be connected two or more in series in one circuit. A less perfect solution is the differential principle introduced by Von Hefner Alteneck, where the difference between the attractions of a series and a shunt-solenoid maintains constant, not the product ei, but the difference $e-i$.

The only perfect solution of the problem is a feeding mechanism which, by combining in itself a shunt-coil and a series-coil, shall keep the product ei a constant, however either factor may vary. All the commercial arc-lamps for lighting in series have shunt-circuits to control the feeding mechanism; though often the arrangement takes the form of a shunt-coil wound (differentially) outside the series-coil of the striking mechanism; so that feeding is accomplished by the shunt-coil demagnetizing the striking electromagnet and momentarily un-striking the arc.

Returning to the Duboscq lamp, it may be observed that, as it possesses no shunt-coil, it can only feed by a weakening of the current in the main circuit. Hence it is obvious that a

Duboscq lamp cannot possibly work in a constant-current circuit. Also two Duboscq lamps will not work in series with one another, as their individual feeding is not independent of the other. Neither will two work in parallel with one another; for the weakening of current in one throws more current through the other, and the instability before alluded to—called "hunting" by electric engineers—becomes yet more pronounced.

The lamp that I have adapted to the Duboscq lantern is one known in commerce as the "Belfast" arc-lamp, its principles of construction being due to Mr. F. M. Newton; but I have had the design altered to suit the special work. In this lamp, as previously mentioned, the striking and feeding mechanisms are separate. The arc is struck by means of an electromagnet E of the tubular pattern, having as its armature an iron disk A, which, when no current is passing, is held up by a short spiral spring at about 3 millim. from the end of the electromagnet. The lower carbon-holder is mounted upon this disk, so that the arc is struck by the downward movement of the lower carbon. The feeding mechanism is both simple and effective. The upper carbon-holder is a long straight tube of brass: it passes through a collar in the frame of the lamp, and also through a metal box B above. This metal box contains a piece of curry-comb with the steel bristles of the comb set to point obliquely inwards and downwards. They grip the carbon-holder and allow it to be pushed downwards, but not upwards. The box itself is mounted upon a strong brass lever, L, close to the point of the lever. One end of this lever is drawn downwards by an adjustable spiral spring S, whilst the other carries an iron armature which stands immediately above the poles of an electromagnet, which is wound with fine wire and placed as a shunt to the lamp. Above the lever there is a contact-screw, platinum-tipped, making contact with the lever, exactly as in the ordinary trembling electric bell, and the lever and contact-screw are included in the shunt-circuit. The attraction of the shunt-magnet for its armature is opposed by the pull of the spiral spring. Whenever, by reason of the resistance of the arc, a sufficient current flows through the shunt-circuit, the opposing spring is overcome, and the lever is set into vibration like the lever of an electric

bell, but more rapidly. The vibratory motion is thus communicated to the box containing the steel wire comb, which at once, by an action well known in mechanism, wriggles the carbon-holder downwards by innumerable small successive impulses. So soon as the motion of the carbon has reduced the resistance of the arc, the shunt-current diminishes and the feeding action ceases, to recommence when required. It is found best for lantern-purposes to send the current upwards through the lamp, the lower carbon being the positive one. A thick cored carbon of 13 to 15 millim. diameter is preferred, as it gives a good luminous crater and burns slowly. A 10-millim. copper-plated carbon is used for the upper electrode, and it is adjusted so that its centre falls slightly in front of the centre of the lower carbon, thereby causing the crater to send its light forward.

The lamp as used in commerce has no focusing-arrangement. In adapting it to the Duboscq lantern, the frame was made narrow; so that when the inner chimney of the Duboscq lantern was removed, the lamp could be dropped entire down the outer chimney, a metal sleeve of the same diameter as the inner chimney being added to the lamp as a guide. At the bottom of the lamp a gun-metal tube was added, tapped inside with a screw-thread, into which works a steel screw having a small hand-wheel near its lower end and a pointed pivot at the extremity. The lamp slides down the chimney of the lantern until the pivot touches the base-board. When the arc burns down the lower carbon, so that the luminous crater is no longer in the optical focus, a turn given by hand to the wheel suffices to raise it to the proper position; but the lamp will burn for ten minutes without requiring any readjustment on this account. The lamp shown to the Physical Society was constructed by Mr. E. Rousseau, Instructor in the Physical Workshop of the Finsbury Technical College, assisted by Mr. A. D. Raine, now Demonstrator in the City and Guilds Central Institution.

XXVIII. *Note on the Tenacity of Spun Glass.*

By E. Gibson *and* R. A. Gregory *.

It is well known that the tenacity of metallic wires increases as the diameter diminishes, so that very fine wires will carry much larger loads than those obtained by calculation based upon the assumption that the breaking weight varies as the square of the diameter. As glass can be drawn into very fine fibres, we have made some observations on the tenacity of this material, comparing the strength of very thin threads with that of rods made from the same glass, but of much greater thickness.

The experiments were carried out in the course of our work in the Physical Laboratory of the Normal School of Science and Royal School of Mines.

In dealing with a substance so brittle as glass, it is evident that special care must be taken to ensure that the observation is not vitiated by rupture due to a shearing stress, at or near the points of support. Precautions were taken to prevent this in all cases, and no experiments are quoted in this paper in which rupture took place near the points of support, or of attachment of the weight.

Three different thicknesses of glass were subjected to experiment: viz., fibres the diameters of which were about 0·002 and 0·004 centim. respectively, and rods with diameters varying between 0·05 and 0·09 centim.

The fibres were attached at the ends of two strips of paper by means of shellac varnish; this on setting was found sufficiently strong to carry more than the breaking weight, without allowing the fibre to slip. A small paper basket suspended from the lower strip carried the load, consisting of fine shot and silica, the latter being added when the fibre was near its breaking-point.

The diameter of the thread was measured at the place of rupture by means of a Compound Microscope with micrometer eye-piece. From data thus obtained the tenacity was calculated with the following results:—

* Read February 12, 1887.

Diameter, in centims.	B. weight in grs.	Tenacity, in dynes per sq. centim.
0·00186	11·76	424×10^7
0·00159	8·70	425×10^7
0·00315	32·26	405×10^7
0·00340	43·23	466×10^7

Some observations were next made on rods about 1 millim. in diameter; the method of support and the loading being changed. Two pieces of angle brass, each about 8 inches long, were substituted for the slips of paper. Through a hole drilled near the end of the angles, a piece of $\frac{1}{8}''$ wire was passed, turned up and soldered to the back. The free extremities of the wires were plaited into rings, which served to support the load and suspend the whole from a hook above.

The ends of the rod were laid in the angles, leaving the glass free for about 12 inches. Small pieces of red-ochre cement (a compound consisting of resin, red ochre, and bees-wax) were placed at intervals along the glass, and a Bunsen flame applied. The cement speedily melted, and imbedded the glass; on cooling, the whole was suspended vertically. A bottle was hung on the wire attached to the lower angle-piece, into which a fine stream of mercury flowed from a reservoir above. The apparatus was so arranged that when the rod broke mercury would no longer fall into the bottle.

The mode of measuring the diameters of the rods differed from that adopted in the case of the fibres. About half an inch of rod was broken away at the place of rupture, and mounted in wax on a piece of looking-glass, the broken section being upwards. Its diameter was then measured by means of a microscope-cathetometer, and the tenacity found as in the case of fibres. The following are the results of four experiments:—

Diameter, in centims.	Weight, in grs.	Tenacity, in dynes per sq. centim.
0·090	3908	60×10^7
0·082	4443	83×10^7
0·050	1948	97×10^7
0·042	1781	126×10^7

These observations show, in the first place, that the tenacity

of fine fibres is very considerably greater than that of thick rods, and that the strength of rods increases as the diameter diminishes. It may be interesting to point out that the tenacity of glass fibres studied by us is nearly as great as that assigned by Wertheim to many of the metals; *e. g.*, the tenacity given by him for annealed steel wire 1 millim. in diameter is 499×10^7 cent.-dynes, and even in the case of drawn steel the tenacity is not greater than twice that of a glass fibre, viz. 998×10^7 cent.-dynes.

With steel pianoforte-wire the tenacity is, however, considerably greater; according to Sir William Thomson (Art. 'Elasticity,' *Encyc. Brit.*, new edition) the breaking-stress is

	Cent.-dynes.
Best pianoforte steel-wire	2318×10^7

The question as to what is the most probable cause of this increase in strength as the diameter diminishes presents some difficulty.

Quincke (*Sitzber. d. Ber. Acad.* 1868, p. 132) has suggested that the great increase observed in the case of metals is due to a surface tension, analogous to that observed in liquids. If this were the true explanation, the breaking-weight could be expressed by the sum of two terms which vary as the diameter and the square of the diameter respectively. This suggestion does not receive much support from our observations, as the results cannot be satisfactorily expressed by means of such a formula. It is, perhaps, more probable that the heating and rapid cooling undergone by the glass when it is drawn out into a fine fibre produces an increase in tenacity; and it is at all events certain that no comparisons can be made between the strengths of different materials unless they have undergone similar treatment, and unless the sizes of the rods or wires submitted to experiment are the same.

XXIX. *On Evaporation and Dissociation.*—Part VI.* *On the Continuous Transition from the Liquid to the Gaseous State of Matter at all Temperatures.* *By* WILLIAM RAMSAY, *Ph.D.*, *and* SYDNEY YOUNG, *D.Sc.*†

[Plates IV., V., VI., & VII.]

IT was proved by Boyle, in 1662, that the volume of a gas, provided temperature be kept constant, varies inversely as the pressure to which it is subjected; this relation may be expressed by the equation $p=\frac{c}{v}$, or pv = constant, where p and v respectively stand for pressure and volume. But subsequent experiments by Van Marum, Oersted, Despretz, and others showed that certain gases do not obey this law; and it is now well known that Boyle's statement is only approximate; for it has been proved by experiment by Regnault, Natterer, and more recently by Amagat, that no gas, under high pressures, is diminished in volume in inverse ratio to the rise of pressure. Indeed Boyle's law could hold only on the assumption that the actual molecules of matter possess no extension in space and exert no attraction on each other. A gas, such as hydrogen, at low pressures, and consequently at large volumes, fills a space very great when compared with the space occupied by the actual molecules; and these molecules are comparatively so distant from one another, that the attraction which they mutually exercise is inappreciable. But, on compression, the actual space occupied by the molecules bears an increased ratio to the space which they inhabit; and, by their approach, the attraction which they exert is also increased. The gas, then, deviates appreciably from Boyle's law.

Gay-Lussac, in 1808, enunciated the law that the volumes of all gases increase by a constant fraction of their volume at

* Parts I. and II., Philosophical Transactions, part i. 1886, pp. 71 and 123; Part III., ibid. part ii., 1886, p. 1; Part IV., Trans. Chem. Soc. 1886, p. 790; Part V., in the hands of the Royal Society.

† Read February 26, 1887.

0° for each rise of 1° in temperature. It was subsequently ascertained by Magnus, and confirmed by Regnault, that certain gases deviate from this law, expanding more rapidly than others. Such gases, as a rule, are at temperatures not far removed from those at which they condense to liquids; that is, their volumes are comparatively small, and the actual size of the molecules and their mutual cohesion begin to manifest themselves within the range of experimental observation. Again, it is evident that no gas can perfectly follow Gay-Lussac's law; but the larger the volume it occupies the smaller is the influence of the disturbing factors. The usual expression for Gay-Lussac's law is $v=c(1+\alpha t)$, or, if the absolute temperature-scale be employed, $v=c\mathrm{T}$.

As a deduction from these laws, it follows that if the volume of unit mass of a gas, supposed to follow them rigorously, be kept constant, the pressure varies directly as the absolute temperature; or $p=c\mathrm{T}$.

Now, so long as the volume of unit mass of a gas is kept constant, the average distance of its molecules from one another will remain constant; and it is a fair assumption that the attraction of the molecules for each other will not vary. It may, of course, be the case that the effect of a rise of temperature on any individual molecule is to alter its actual volume; but of this we know nothing; and, in default of knowledge, it has been assumed by us that no such alteration takes place. If these assumptions are correct, it follows that the temperature and pressure of gases—and indeed the same assumptions may be extended to liquids—should then bear a simple relation to each other. We have obtained experimental proof of a convincing nature that this is the case; and in a preliminary note to the Royal Society, read on January 6, we promised such a proof. This proof is the subject of the present paper; and we must ask for indulgence in quoting a large array of figures, some of which have already been published, on the ground that such an important generalization requires as much experimental evidence as can be brought to bear on it.

The relation between the pressures and temperatures of a

liquid or a gas at constant volume is expressed by the equation

$$p = bT - a ;$$

where p is the pressure in millimetres, T the absolute temperature, and b and a constants. The values of these constants depend on the nature of the substance and on the volume. It follows from this, that if a diagram be constructed to express the relations of pressure, temperature, and volume of liquids and gases, where pressure and temperature form the ordinates and abscissæ, the lines of equal volume are straight*.

We have proved this to be the case for ethyl oxide (ether) between the temperatures 100° and 280°, and for volumes varying from 1·85 cubic centim. per gram to 300 cubic centim. per gram. This proof we now proceed to give.

The data for the calculations are at present in the press, and will shortly appear in the Philosophical Transactions for 1886, p. 10. A diagram (which will accompany that memoir) was constructed with the greatest care, showing isothermal lines, the ordinates and abscissæ being respectively pressures and volumes. It was possible to read pressure accurately to within 20 millim.; and volume, up to a volume of 3·1, to within 0·001 cubic centim. per gram; and, at volumes greater than 3·1 cubic centim. per gram, to 0·01 cubic centim. per gram. Pressures corresponding to each isothermal were then read off on the equal volume-lines, from curves constructed to fit the experimental points as accurately as could be drawn with the help of engineers' curves. These pressures and temperatures were then mapped as ordinates and abscissæ; and it was found that points corresponding to each volume lay in a straight line. Again, two points were chosen on these equal volume-lines, as far apart as the scale of the diagram would permit, and the values of the change of pressure per unit change of temperature, $\frac{dp}{dt}$, were ascertained for each

* Amagat (*Comptes Rendus*, xciv. p. 847) has stated a similar relation for gases; his data are, however, imperfect, and he expressly states that the law does not apply to liquids.

separate volume chosen. To eliminate irregularities, these values were smoothed graphically; but it was difficult to find any very satisfactory method. The method employed for ether, which we found to give the best results, was to map as ordinates the ratios between these values, and similar values calculated on the supposition that the gas or liquid followed the usual gaseous laws, against the reciprocals of the volumes as abscissæ. A curve was then drawn, taking a mean course among the actual points, and the values of $\frac{dp}{dt}$ were calculated from readings at definite volumes. This expression, $\frac{dp}{dt}$, is the b of our formula. Having thus obtained the most probable value of b for each volume, the value of a at each volume was ascertained by calculation from each individual point read from the original curves, and at each volume the mean of all was chosen.

Isothermals were then calculated by means of the equation $p=b\mathrm{T}-a$, T being kept constant; and those values of a and b corresponding to the volumes required being selected. These calculated isothermals are shown on Plate IV.; and the lines of equal volume, or isochors*, on Plate V. It is evident, from inspection of the former, that the calculated lines correspond as closely as possible with the actual observations.

It is necessary now to give the data on which these deductions are based. The following Table gives those points corresponding to lines of equal volume read from the diagram constructed from experimental observations.

* From ἴσος, *equal*, and χωρεῖν, *to contain*. Another suitable word would be "*isoplethe*," but we have Professor Jowett's preference for the one selected. Either of these terms seems preferable to that (*isometrics*) already proposed.

TABLE I.

Volume. cub. cent.									
1·9	P.... 12340	19500							
	T.... 145°·8	150°							
2·05	P.... 16500	35700							
	T.... 162°·7	175°							
2·10	P.... 17700	29850							
	T.... 166°·9	175°							
2·15	P.... 18700	25260	38620						
	T.... 170°·2	175°	185°						
2·20	P.... 19560	22160	34160						
	T.... 173°·05	175°	185°						
2·25	P.... 20450	31070	36960	39750					
	T.... 175°·9	185°	190°	192°					
2·3	P.... 21250	28680	34000	36430	38600	40200			
	T.... 178°·35	185°	190°	192°	193°·8	195°			
2·4	P.... 25250	30380	32320	33980	35300	37500	40320		
	T.... 185°	190°	192°	193°·8	195°	197°	200°		
2·5	P.... 27920	29820	31230	32300	34200	37130	41580		
	T.... 190°	192°	193°·8	195°	197°	200°	205°		
2·75	P.... 26840	27970	28860	30270	32850	36240	40200	47200	
	T.... 192°	193°·8	195°	197°	200°	205°	210°	220°	
3·0	P.... 27270	28090	27900	29250	31230	34150	37440	43620	
	T.... 193°	195°	195°	197°	200°	205°	210°	220°	
3·3	P.... 27800	27740	28640	30550	32950	35960	40900		
	T.... 195°	195°	197°	200°	205°	210°	220°		
3·7	P.... 27640	27720	28470	30100	32240	34700	38840	40100	52700
	T.... 195°	195°	197°	200°	205°	210°	220°	223°	250°

4	P....	27600	27690	28440	29900	31840	34050	37830	38900	50260							
	T....	195°	195°	197°	200°	205°	210°	220°	223°	250°							
5	P....	27000	27460	27460	28150	29250	30800	32400	35440	36140	44620						
	T....	193°·8	195°	195°	197°	200°	205°	210°	220°	223°	250°						
6	P....	26100	26400	26640	27000	26970	27560	28400	29700	30950	33260	33950	40580	49620			
	T....	192°	193°	193°·8	195°	195°	197°	200°	205°	210°	220°	223°	250°	280°·35			
7	P....	25130	25500	25750	25950	26210	26150	26670	27350	28400	29450	31340	31950	37330	44680		
	T....	190°	192°	193°	193°·8	195°	195°	197°	200°	205°	210°	220°	223°	250°	280°·35		
8	P....	23250	23380	24300	24660	24850	25060	25250	25190	25640	26200	27030	27930	29500	30100	34580	40750
	T....	185°	185°	190°	192°	193°	193°·8	195°	195°	197°	200°	205°	210°	220°	223°	250°	280°·35
9	P....	22540	22600	23400	23650	23830	23950	24130	24050	24450	24950	25650	26500	27810	28330	32160	37400
	T....	185°	185°	190°	192°	193°	193°·8	195°	195°	197°	200°	205°	210°	220°	223°	250°	280°·35
10	P....	21650	21790	22350	22650	22770	22870	23030	23000	23350	24400	26240	26670	30000	34590		
	T....	185°	185°	190°	192°	193°	193°·8	195°	195°	197°	205°	220°	223°	250°	280°·35		
11	P....	19550	19550	20800	20860	21450	21700	21800	21870	22020	21990	22270	23200	24820	25200	28170	32200
	T....	175°	175°	185°	185°	190°	192°	193°	193°·8	195°	195°	197°	205°	220°	223°	250°	280°·35
12	P....	18780	18800	19950	19990	20530	20740	20850	20930	21060	20990	22100	23540	23870	26510	30050	
	T....	175°	175°	185°	185°	190°	192°	193°	193°·8	195°	195°	205°	220°	223°	250°	280°·35	
13	P....	18050	18070	19130	19090	19650	19840	19960	20030	20130	20020	21100	22340	22740	25020	28200	
	T....	175°	175°	185°	185°	190°	192°	193°	193°·8	195°	195°	205°	220°	223°	250°	280°·35	
14	P....	17330	17400	18300	18300	18740	18960	19100	19200	19250	19190	20250	21240	21730	23770	26600	
	T....	175°	175°	185°	185°	190°	192°	193°	193°·8	195°	195°	205°	220°	223°	250°	280°·35	
15	P....	16670	16750	17570	17570	17970	18280	18500	18380	20250	22580	25190					
	T....	175°	175°	185°	185°	190°	193°·8	195°	195°	220°	250°	280°·35					
16	P....	16000	16100	16830	16850	17180	17500	17740	17610	19380	21460	23880					
	T....	175°	175°	185°	185°	190°	193°·8	195°	195°	220°	250°	280°·35					
17	P....	15440	15520	16180	16200	16530	16840	17030	16890	18540	20470	22730					
	T....	175°	175°	185°	185°	190°	193°·8	195°	195°	220°	250°	280°·35					

Table I. (*continued*).

Volume. cub. cent.												
18	P....	13350	14880	14940	15580	15600	15900	16270	16330	16210	17780	19600	21640
	T....	150°	175°	175°	185°	185°	190°	193°·8	195°	195°	220°	250°	280°·35
19	P....	12840	14370	14390	15020	15020	15330	15570	15720	15610	17040	18780	20670
	T....	150°	175°	175°	185°	185°	190°	193°·8	195°	195°	220°	250°	280°·35
20	P....	12240	13880	13880	14500	14480	14790	15000	15130	15020	16400	18000	19800
	T....	150°	175°	175°	185°	185°	190°	193°·8	195°	195°	220°	250°	280°·35
25	P....	10700	11800	11860	12280	12330	12500	12730	12800	12760	13820	14960	16440
	T....	150°	175°	175°	185°	185°	190°	193°·8	195°	195°	220°	250°	280°·35
30	P....	9350	10220	10600	10780	10960	11050	11970					
	T....	150°	175°	185°	190°	193°·8	195°	223°					
40	P....	7430	8030	8340	8460	8560	8600	9330					
	T....	150°	175°	185°	190°	193°·8	195°	223°					
50	P....	6150	6620	6840	6940	7010	7020	7600					
	T....	150°	175°	185°	190°	193°·8	195°	223°					
75	P....	3650	4290	4600	4730	4800	4860	5210					
	T....	100°	150°	175°	185°	190°	195°	223°					
100	P....	2820	3280	3500	3600	3680	3710	3950					
	T....	100°	150°	175°	185°	190°	195°	223°					
150	P....	1695	2509										
	T....	59°·7	195°										
200	P....	1260	1897										
	T....	50°	195°										
250	P....	1020	1525										
	T....	50°	195°										
300	P....	860	1275	1368									
	T....	50°	195°	223°									

Lines of equal volume, or isochors, were then mapped from these results, pressures and temperatures being ordinates and abscissæ. The values of $\frac{dp}{dt}$, or b, were then read for each separate volume and smoothed, as before described. The following Table shows the read values, and the values after smoothing. The values of a are also given, calculated by the equation $a=bt-p$, from the results shown on the previous table; the means of all the individual results for each volume are stated.

TABLE II.

Volume.	b found.	b calculated.	log b.	a.
1·85	.....	2034	3·30826	826860
1·90	1746	1861	3·26986	767670
1·95	.....	1716	3·23452	715860
2·0	.....	1597	3·20335	672820
2·05	1561	1492	3·17372	633070
2·1	1500	1405	3·14777	600110
2·15	1342	1320	3·12061	566170
2·2	1211	1243	3·09462	535100
2·25	1155	1175	3·07022	507170
2·3	1117	1115	3·04744	482160
2·4	1020	1010	3·00423	437240
2·5	920·8	919·7	2·69363	397970
2·75	732·0	732·0	2·86451	313605
3·0	623·5	621·7	2·79357	262917
3·3	535·7	532·8	2·72659	221630
3·7	453·1	454·6	2·65762	185109
4·0	413·7	413·7	2·61669	165996
5·0	321·5	319·1	2·50392	121895
6·0	257·15	254·2	2·40511	91906
7	213·6	208·7	2·31959	71464
8	178·0	176·1	2·24584	57203
9	150·95	151·4	2·18022	46742
10	130·8	132·7	2·12280	39079
11	116·25	117·6	2·07027	33037
12	104·45	105·5	2·02336	28401
13	94·29	95·54	1·98017	24659
14	85·00	87·09	1·93998	21567
15	78·82	80·06	1·90339	19125
16	72·29	73·95	1·86893	17049
17	67·53	68·76	1·83733	15313
18	62·53	64·24	1·80780	13854
19	59·23	60·11	1·77893	12533
20	55·70	56·43	1·75151	11386
25	43·75	43·26	1·63612	7529
30	35·60	34·99	1·54393	5412
40	26·23	25·09	1·39945	3159
50	20·38	19·46	1·28925	2077
75	12·82	12·50	1·09674	994
100	9·32	9·119	0·95997	571
150	5·87	5·923	0·77254	270
200	4·38	4·396	0·64306	160·5
250	3·46	3·483	0·54198	105
300	2·93	2·858	0·45605	59

Table III.

V.	0°	100°	150°	160°	175°	185°	190°	192°	193°·8	195°	197°	200°	205°	210°	220°	223°	250°	280°
	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.
1·85	−271700	−68330	+33340	+53680														
1·9	−259490	−73330	19740	38360														
1·95	−247390	−75790	10110	27180														
2·0	−236800	−77070	2780	18760	42720													
2·05	−225800	−76710	−2020	12900	35270													
2·1	−216460	−75930	−5670	8390	29470	43530												
2·15	−205780	−73770	−7660	5440	25240	38450	45040	47680	50060									
2·2	−195650	−71300	−9130	3310	21960	34400	40610	43090	45330									
2·25	−186260	−68710	−9940	2880	19450	31210	37080	39430	41550	42970								
2·3	−177650	−66110	−10340	820	17550	28710	34280	36510	38520	39860	42090							
2·4	−161750	−60590	−10100	0	15150	25250	30290	32310	34130	35340	37360	40390						
2·5	−146900	−54930	−8950	250	14040	23240	27830	29670	31330	32440	34280	37030	41630	46230				
2·75	−113700	−40490	−3900	3429	14410	21730	25380	26850	28170	29050	30510	32710	36370	40030	47350	49540		
3·0	−93135	−30965	+ 115	6335	15665	21875	24985	26225	27345	28095	29335	31205	34315	37425	43635	45495		
3·3	−76170	−22880	3760	9080	17080	22410	25070	26130	27100	27740	28800	30400	33070	35730	41060	42650		
3·7	−61110	−15550	7180	11730	18550	23090	25360	26270	27090	27640	28550	29910	32190	34460	39060	40370	52640	

4	−53056	−11686	9004	13144	19344	23484	25544	26374	27124	27624	28444	29684	31754	33824	37964	39204	50374	
5	−34783	−2875	13085	16275	21065	24255	25845	26485	27055	27445	28085	29035	30635	32225	35425	36375	44995	54565
6	−22520	+2897	15604	18144	21954	24504	25774	26274	26734	27044	27554	28314	29584	30854	33394	34164	41024	48644
7	−14480	6393	16830	18918	22049	24136	25179	25596	25972	26224	26636	27266	28311	29356	31446	32061	37706	43966
8	−9119	8495	17301	19063	21705	23467	24346	24698	25016	25228	25580	26108	26989	27870	29641	30158	34914	40200
9	−5481	9743	17314	18829	21100	22615	23371	23674	23947	24129	24432	24886	25643	26401	27915	28369	32457	37001
10	−2858	10410	17044	18371	20361	21688	22351	22616	22855	23015	23280	23678	24342	25005	26332	26729	30311	34293
11	− 943	10814	16692	17868	19631	20807	21394	21629	21841	21983					24922	25274	28448	31976
12	+ 407	10960	16237	17292	18875	19930	20457	20668	20859	20986					23624	23940	26789	29956
13	1422	10976	15753	16709	18142	19097	19574	19765	19937	20053					22441	22727	25307	28173
14	2209	10929	15273	16144	17451	18322	18757	18931	19088	19193					21370	21631	23982	26596
15	2730	10736	14738	15539	16740	17541	17941	18100	18245	18341					20343	20582	22744	25146
16	3130	10534	14231	14971	16080	16820	17189	17337	17470	17559					19408	19629	21626	23845
17	3458	10334	13772	14460	15491	16179	16522	16660	16784	16867					18586	18791	20648	22711
18	3683	10107	13319	13962	14925	15568	15889	16017	16133	16210					17816	18009	18743	21671
19	3876	9887	12903	13494	14395	14997	15297	15417	15525	15598					17100	17280	18903	20707
20	4019	9662	12478	13048	13895	14459	14741	14854	14955	15023					16434	16606	18127	19820
25	4282	9608	10771	11204	11853	12286	12502	12588	12666	12718					13800	13930	15098	16396
30	4140	7639	9388	9738	10263	10613	10788	10858	10921	10963					11838	11942	12887	13937
40	3690	6199	7453	7704	8080	8331	8456	8506	8552	8582						9284		
50	3237	5183	6227	6351	6643	6838	6935	6974	7009	7033						7578		
75	2417	3667	4291	4416	4604	4729	4791	4816	4839	4854						5244		
100	1919	2831	3287	3378	3515	3606	3651	3670	3686	3697						3952		

The lines of equal volume calculated from these values of a and b are reproduced graphically in Plates V. and VI. We propose to discuss the form of the diagram later.

We have calculated the values of p for various isothermals at the above volumes. These are shown in Plate IV.; the actual experimental observations, of which a detailed account is given in the 'Philosophical Transactions,' 1886, p. 10 *et seq.*, are represented by circles. It is evident that the curves through the calculated points represent the actual measurements very closely, indeed as nearly as unavoidable error of experiment allows. It is to be noticed that the greatest divergence is at the temperatures 250° and 280°, but the deviations are in opposite directions, and must therefore be ascribed to experimental error.

Table III. (pp. 202, 203) gives the data from which the calculated isothermals were constructed.

As volumes above 30 cubic centim. per gram are not given in the diagram, we have thought it advisable to show the correspondence of calculated and observed results by a table; the calculated numbers were read from curves specially constructed from the formula $p=bt-a$, and the observed results are those actually furnished by our experiments. It will be seen that the correspondence is very close.

Table III. (*continued*).

Volume.	Pressure found.	Pres. calculated.	Volume.	Pressure found.	Pres. calculated.
c. c.	millim.	millim.	c. c.	millim.	millim.
Temperature 100°.					
54·06	4817	4875	74·42	3668	3685
55·50	4720	4760	77·34	3546	3550
56·95	4627	4670	83·14	3334	3320
59·84	4434	4470	86·02	3243	3215
62·73	4265	4290	88·89	3150	3120
65·63	4102	4100	94·63	2978	2970
68·56	3946	3970	97·51	2893	2900
71·48	3818	3815			
Temperature 150°.					
31·09	9066	9110	68·64	4651	4675
33·94	8497	8520	74·51	4304	4310
39·66	7484	7505	80·35	4027	4010
45·43	6674	6745	86·13	3783	3760
51·23	6024	6100	91·87	3564	3530
57·02	5480	5560	97·63	3375	3345
62·81	5034	5090			

Table III. (*continued*).

Temperature 175°.					
Volume.	Pressure found.	Pres. calculated.	Volume.	Pressure found.	Pres. calculated.
c. c.	millim.	millim.	c. c.	millim.	millim.
31·11	9906	9975	68·69	4987	5000
33·96	9248	9290	74·56	4626	4630
39·68	8065	8130	80·40	4307	4310
45·46	7208	7205	86·18	4035	4030
51·26	6485	6510	91·93	3803	3800
57·06	5903	5940	97·69	3589	3600
62·85	5396	5450			
Temp. 185°.			Temp. 190°.		
31·12	10284	10300	31·12	10455	10460
45·47	7434	7460	45·47	7549	7545
59·97	5811	5865	59·97	5902	5940
74·58	4755	4770	74·59	4828	4810
86·21	4159	4140	86·21	4210	4190
91·95	3915	3900			
97·72	3692	3690			
Temp. 192°.			Temp. 193°·8.		
31·12	10544	10520	31·12	10587	10590
45·48	7590	7600	45·48	7627	7630
59·98	5930	5970	59·98	5951	6000
74·60	4847	4850	74·60	4867	4875
86·42	4230	4200	86·42	4252	4220
Temp. 195°.			Temp. 197°.		
31·12	10631	10620	33·17	10108	10055
45·48	7651	7675	38·07	8972	8965
59·98	5969	6020	47·84	7312	7320
74·60	4884	4895	57·59	6166	6130
86·42	4260	4230	67·33	5356	5340
91·97	4007	4000	77·05	4718	4735
97·74	3797	3775	86·75	4219	4220
			96·44	3820	3820
Temperature 223°·25.					
31·15	11567	11550	80·50	4870	4880
45·51	8260	8240	86·29	4556	4550
60·02	6412	6420	92·04	4289	4270
74·65	5232	5225	97·81	4044	4020

These complete the data for ether. It appeared necessary

to examine the relations between volume, temperature, and pressure for carbon dioxide, because it is chemically entirely different from ether, and also because the data are furnished by Dr. Andrews, whose experimental skill was very great; and we shall prove that his results entirely corroborate our views.

It was first necessary to correct the pressures given by him in atmospheres by means of Amagat's results, so far as that is possible. As Amagat's experiments on the compressibility of dry air do not extend beyond 65,000 millim., no correction was possible above that pressure; and extrapolation is inadmissible inasmuch as the minimum value of pv for air is at 60,000 millim.

Data for obtaining the actual volume of carbon dioxide used are given by Andrews. The weight was calculated in the following manner:—Andrews gives the volume of carbon dioxide filling his tubes at 0° and 076 millim., and from Regnault's data the weight was calculated. This refers to Andrews's first paper (Phil. Trans. 1869, ii. p. 575). In his second paper (Phil. Trans. 1876, p. 421) no direct data are given from which the weight can be determined; but we succeeded, by combining the results given in his various tables, in arriving at the weight without any serious error. His results are reproduced in an available form in the following Tables.

TABLE IV. (The first quantity weighed 0·000612 gram.)

Temp. 13°·1.		Temp. 21°·5.		Temp. 31°·1.	
P.	Vol. of 1 gram.	P.	Vol. of 1 gram.	P.	Vol. of 1 gram.
millim.	c. c.	millim.	c. c.	millim.	c. c.
68726 *	1·038	46600	1·232	63462	1·389
56333	1·059	46383	1·241	61416	1·425
40725	1·104	45490	1·484	59540	1·470
37631	1·124	45155	2·270	57847	1·495
37459	1·145	44962	3·124	56178	1·812
37074	1·377	44787	4·760	55010	2·234
36942	1·547	34907	8 118	54588	2·338
36816	1·972			53089	3·814
36719	2·758			51709	4·192
36668	3·733			50390	4·534
36610	5·004			49118	4·855
36528	6·554			47910	5·147
36433	6·595			46725	5·471
35497	6·964			45622	5·750
				44577	6·009
				43611	6·265
				43182	6·515
				41761	6·765
				40895	7·003

* The pressures to which an asterisk is affixed are not corrected.

Table IV. (*continued*).

Temp. 32°·5.		Temp. 35°·5.		Temp. 48°·1.	
P.	Vol. of 1 gram.	P.	Vol. of 1 gram.	P.	Vol. of 1 gram.
millim.	c. c.	millim.	c. c.	millim.	c. c.
63246	1·461	81775 *	1·330	83144 *	1·999
59425	1·612	75673 *	1·392	72344 *	2·999
58501	1·818	70406 *	1·476	62837	4·061
56813	2·954	68035 *	1·532	56311	5·057
55151	3·543	64516	1·624	51012	5·991
54839	3·629	60552	2·511	46675	6·887
53292	4·034	57057	2·549		
42808	6·595	53977	4·233		
		51166	4·831		
		48625	5·396		
		46340	5·935		
		44263	6·432		
		42383	6·917		

Table IV. (*continued*).

(The weight of the second quantity was 0·0018075 gram.)

Temp. 0°.		Temp. 6°–6°·9.		Temp. 63° 6–64°.		Temp. 99°·5–100°·7.	
P.	Vol. of 1 gram.	P.	Vol. of 1 gram.	P.	Vol. of 1 gram.	P.	Vol. of 1 gram.
millim.	c. c.	millim.	c. c.	millim.	c. c.	millim.	c. c.
25865	10·53	25870	11·25	169420*	1·401	169910 *	1·816
23325	12·36	23291	13·09	110610*	1·907	110530 *	3·161
20800	14·53	20822	15·16	81229*	3·363	80323 *	5·057
18674	16·77	18681	17·11	60422	5·477	59985	7·192
15155	21·73	16777	19·90	48425	7·485	47910	9·391
12277	27·83	15161	22·54	40554	9·462	40169	11·52
9081	39·09	12907	27·16	34806	11·45	34382	13·72
		11093	32·18	30351	13·48	29910	15·96
		9994	36·12	26212	15·99	25930	18·74
		9087	40·06	23568	18·00	23325	21·03
				21106	20·41	20876	23·70
				18866	23·08	18711	26·65
				17000	25·86	16858	29·74
				15356	28·88	15214	33·09
				13288	33·74	13153	38·58

As with ether, these numbers were plotted graphically up to a pressure of 75,000 millim. Above 65,000 millim. no true correction for deviation of Andrews's air-gauge was possible;

* The pressures to which an asterisk is affixed are not corrected.

but approximate corrections were introduced. It was possible to read pressures to within 30 millim. on the scale employed, and volume to within 0·02 cub. centim. per gram. Pressures corresponding to even volumes were read off, as with ether; and on mapping the isochors with temperatures as abscissæ and pressures as ordinates, the resulting points lay in straight lines. The values of b, i. e. of $\frac{dp}{dt}$, were then read off, and smoothed, by mapping them against the reciprocals of the volumes. After smoothing, the values of a were calculated at each volume, making use of the pressures previously read from the curve representing isothermals. The diagram (1) on Plate VII. was then constructed from these smoothed values. The crosses denote our readings of pressures at the temperatures chosen by Andrews for his isothermals. These values of a and b were then made use of in recalculating isothermals at the above temperatures, and the diagram (2) on Plate VII. represents the curves complete so far as Andrews's data allow. The circles represent Andrews's actual measurements; and it is evident that no better concordance could be expected. The tables which follow give the data afforded by Andrews's experiments.

Table V.

Pressures read from Curves originally drawn from Andrews's experimental data, and represented by circles in the diagram (2), Plate VII.

Vol.	Temperature.									
	0°.	6°.	13°·1.	21°·5.	31°·1.	32°·5.	35°·5.	48°·1.	64°.	100°.
c. c.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.
30	11430	11790	...	...	...	...	...	...	14820	16710
25	13480	13830	...	...	...	...	...	...	17500	19860
20	16200	16570	...	...	...	...	...	...	21450	24300
15	20300	20970	...	...	...	...	...	...	27700	31570
12	23730	24800	...	...	...	...	...	...	33500	38730
10	26715	28000	...	...	...	...	...	...	38970	45330
8	...	...	...	35340	37560?	37800?	38400?	41930	46200	54630
7	...	...	35340	38220?	40970	41370	42060	46200	50850	61500
6	...	...	...	41100?	44700	45060	46000	51000	56700	
5	...	...	...	44100?	48600	49200	50370	56760		
4·5	...	...	...	...	50550	51400	52830	60000		
4·0	...	...	...	...	52420	53430	55140	63500		
3·5	...	...	...	...	54250	55260	57240	67000		
3·0	...	...	...	...	55800	56700	59130	72400		

Table VI.

Read and Smoothed Values of b, and Values of a.

Vol.	b, read.	b, smoothed.	log b.	a.
c. c.				
30	52·3	52·5	1·72016	2877
25	63·3	64·0	1·80618	4024
20	81·9	82·0	1·91381	6256
15	113·85	114·5	2·05881	10990
12	150·0	149·9	2·17580	17103
		188·5	2·27531	24718
8	255·0	252·1	2·40175	39120
7	300·0	302·0	2·48001	50970
6	368·1	373·0	2·57171	68877
5	472·3	475·5	2·67715	96008
4·5	553·7	548·5	2·73918	116230
4·0	654·0	638·0	2·80482	141525
3·5	750·0	759·5	2·88053	176860
3·0	933·6	936·5	2·97151	229420

Table VII.

Calculated Pressures on Isothermal Curves, at definite volumes.

Vol.	Temperature.									
	0°.	6°.	13°·1.	21°·5.	31°·1.	32°·5	35°·5.	48°·1.	64°.	100°.
c. c.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.
30	11456	11771	...	...	13088	...	...	13981	14816	16706
25	13448	13832	...	...	15439	...	...	16526	17544	19848
20	16130	16622	...	...	18680	...	..	20074	21378	24330
15	20269	20956	...	...	23830	...	...	25776	27597	31719
12	23719	24719	25783	27043	28482	...	29140	31030	33413	38810
10	26742	27873	29212	30796	32605	...	33433	35809	38806	45592
8	29731	31244	33036	35155	37576	37928	38684	41862	45872	54952
7	31476	33288	35433	37971	40870	41291	42197	46003	50800	61680
6	32973	35190	37843	40973	44553	45073	46193	50893	56820	70250
5	33802	36652	40032	44032	48592	49262	50682	56672	64232	
4·5	33510	36800	40700	45310	50570	51340	52980	59890	68620	
4·0	32645	36475	41005	46365	52495	53385	55295	63335	73485	
3·5	30480	35040	40440	46820	54110	55170	57440	67020		
3·0	26240	31860	38520	46380	55370	56680	59620	71290		

It will be seen that the highest calculated pressure is about 73,500 millim. Andrews gives measurements at much higher pressures; but these are few in number and uncertain, and the correction for the compressibility of air is moreover unknown. Hence it was impossible to make use of them in determining the values of b.

On reference to Andrews's paper (Phil. Trans. 1876, p. 435)

it will be seen that he compared the relation of increase of pressure to temperature-difference at constant volumes, and came to a conclusion opposed to ours. This is owing to his having made very few observations, and having accidentally chosen those which support his statement. If the coefficients of increase of pressure for unit rise of temperature be calculated by means of Table V., it may be noticed that, although irregular, there is no tendency towards a rise or fall of the coefficient.

Regnault has measured the rise of pressure of gaseous carbonic anhydride at constant volume. He gives the results of four experiments, none of which are available for our purpose, inasmuch as the volumes of a gram are too large.

Reverting to the behaviour of ether, as shown on Plate IV., it will be seen that the curves have been drawn in the region where measurements are impossible. These curves have all the same general form. After rise of pressure and decrease of volume have proceeded for some distance, the curves bend downward, presenting the abnormal feature of decrease of volume with fall of pressure. The pressure continues thus to fall, and at 160° the isothermal touches zero-pressure. At lower temperatures, with small volumes, the pressure becomes negative, and may even represent an enormous tension. At 0° the isothermal at vol. 1·85 cub. centim. per gram reaches the almost incredibly great tension of −271,700 millim.; and it has at that volume (the smallest our results allow us to calculate) by no means reached its limit. At still smaller volumes the tension would doubtless still increase, until the curve turned, and further decrease of volume would be represented, as it is at higher temperatures, by increase of pressure.

The existence of these unrealizable portions of such isothermal curves was, we believe, first suggested by Prof. James Thomson, in a paper in the 'Proceedings of the Belfast Natural History and Philosophical Society,' Nov. 29, 1871. Since that time attempts have been made to express relations between the pressure, temperature, and volume of gases and liquids by Van der Waals and by Clausius; and the formulæ which they propose, and which we hope to consider in a subsequent paper, give isothermals of similar form. Portions of these curves have, indeed, been experimentally verified. In

Professor Thomson's paper, above referred to, he points out that Donny, Dufour, and others have observed the phenomenon commonly alluded to as "boiling with bumping." This is usually the effect of a rise of temperature at constant pressure. But it may equally well be produced, as we have frequently had occasion to remark, at constant temperature by lowering pressure. If the diagram on plate iii. in our memoir on alcohol (Phil. Trans. 1886, part i. p. 156) be referred to, it will be seen that our actual measurement of such reduced pressure was made on the isothermal 181°·4. Mr. John Aitken, in an extended series of experiments on this subject (Trans. Royal Scott. Soc. of Arts, vol. ix.), has shown that such "superheating" can take place only in absence of a free surface, *i. e.* the existence of gaseous nuclei in the liquid, into which evaporation may take place. And Mr. Aitken has also shown that a gas may be compressed to a volume smaller than that at which liquefaction usually occurs, at any given temperature, without formation of liquid. The space, again, if no nuclei be present on which condensation may take place, remains "supersaturated with vapour." It is evidently, therefore, only the instability of such conditions which prevents their complete realization*.

The formulæ of Clausius and Van der Waals are based on the assumption that two causes are in operation (those referred to in the beginning of this paper), viz. the actual size of the molecules, and their mutual attraction. It is possible, by help of these assumptions, to realize the nature of the continuous change from the gaseous to the liquid state of matter. When a gas at a given temperature is reduced in volume its molecules necessarily approach each other, and their attraction for one another increases. This attraction aids the increase of pressure in reducing volume. When a certain volume is reached, the attraction has become so marked that further reduction of volume is accompanied by fall of

* The reasoning of a recent paper by Wroblewski (*Monatsheft der Chemie*, Wien, July 1886, p. 383) rests on the assumption that such conditions are inconceivable. He supposes lines of equal density to be curves, and on their close approach to the vapour-pressure curve to run parallel with it. His conclusions are therefore not borne out by experimental facts.

pressure. If a certain volume be chosen on the descend portion of an isothermal, a state of balance may be imagi where pressure and cohesion unite in maintaining the volu constant against the kinetic energy of the molecules, tend to cause expansion.

The conception of negative pressure, or tension, is that low temperatures and small volumes the cohesion is such th in order to overcome it and increase volume, it would necessary to apply tension to each molecule. But after lowest pressure or greatest tension has been attained, actual size of the molecules presents a bar to closer approa and to cause further decrease of volume pressure must ag be applied. It is not to be supposed that at any given volu only one of these factors is operative; the actual size of molecules exerts its influence even at large volumes, and t cohesion does not disappear, but no doubt immensely increas as the volume is reduced, even when that reduction requi rise of pressure. Still, a mental picture of the process ma we think, best be attained by directing attention to cohesio when volume is being decreased with fall of pressure, and the influence of the actual size of the molecules when volu is small.

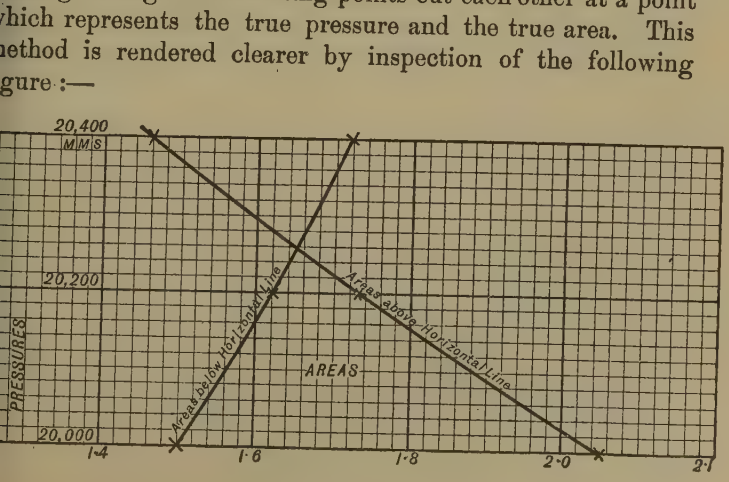
When a liquid is converted into gas, heat is absorbed, work is done on the liquid. We have previously (*loc. ci* given tables showing the heats of vaporization of ether various temperatures. Our experiments have confirmed t prediction that the heat of vaporization of stable liqui decreases with rise of temperature, and in all probabili becomes zero at the critical temperature. Now the volume a fluid may be changed, either keeping the pressure consta or allowing it to vary during the operation; but if the initi pressure and final pressure are the same, the variation pressure during the operation does not affect the total wo done. A liquid may be changed into saturated vapour at a given temperature in the usual manner, when the intermedia states are represented by non-homogeneous mixtures of liqu and saturated vapour. The area enclosed between the vapou pressure line and lines drawn vertically from its terminal point cut by the line of zero pressure (or pressure × change volume), represents graphically the external work performe in evaporating a liquid. If, however, the change of conditio

be not abrupt, but continuous, the area enclosed by the isothermal below the vapour-pressure line must be equal to that above the vapour-pressure line (see Plate IV.). If this were not the case, the amount of work required to effect the continuous change would differ from that required for the abrupt change of state.

Now it is evident that a slight alteration in the position of the vapour-pressure line would have great influence on the relative areas enclosed by the isothermal above and below the vapour-pressure line ; and it may also be seen that, when these areas are rendered equal by a horizontal line, the position of that horizontal line must represent the true vapour-pressure.

We have determined the position of the horizontal line in the following manner :—

Knowing approximately the position of the vapour-pressure line at a given temperature, three pressures were chosen—the highest above and the lowest below the experimentally determined vapour-pressure ; and by means of a planimeter (by Stanley of Holborn) the areas enclosed between each horizontal line and the curves respectively below and above it, formed by the isothermal lines, were measured. To ascertain what position of the horizontal line would render these areas equal, the values of each set of three areas were mapped on rectional paper as abscissæ, the pressures corresponding to the position of the horizontal lines being ordinates. The curves passing through the resulting points cut each other at a point which represents the true pressure and the true area. This method is rendered clearer by inspection of the following figure :—



It is evident that these vapour-pressures depend entirely o measurements represented in the diagram outside the are bounded by the curve representing orthobasic volumes of ga and liquid. It will be seen, on reference to the table on p. 20 that the agreement between these calculated vapour-pressure and those experimentally determined is a very close one, th greatest difference being about 1 per cent. This agreemen between experimentally observed vapour-pressures and thos depending on the formula $p=bt-a$ is very remarkable, an it is difficult to believe that, if the isochoric lines were curve such an agreement could exist.

What are usually termed vapour-pressures, then, are thos pressures at which horizontal lines drawn through them rende the areas enclosed by the isothermal lines below the horizonta lines equal to those above them. But there are other tw conditions of matter, each of which has its characteristi pressures. One of these is represented by the highest pressur attainable on any isothermal, or the summit of the curve abov the vapour-pressure line; and the other the apex of the curv below the vapour-pressure line. Each temperature chosen ha its particular value for each of these conditions; and it i evident that the relations between the temperatures an pressures corresponding to the inferior or reversed apices, a well as those corresponding to the superior apices, would eac form a special curve.

The following Table gives the final results of the calculatio of vapour-pressures by the method of areas; and, for th sake of comparison, the actually found vapour-pressures ar appended. The pressures at the superior and inferior apice of the isothermal curves, and also the enclosed areas, ar given*.

* The areas are in square inches; the scale was 2000 millim. an .2 cub. centim. per gram to the inch. It would be easy, if necessary, t convert these data into actual work.

TABLE VIII.

Temperature.	Vapour-pressures, deduced from areas.	Vapour-pressures, mean of observed.	Pressure, superior apex.	Pressure, inferior apex.	Area above or below vapour-pressure line.
°	millim.	millim.	millim.	millim.	sq. in.
192	26350	26331	26490	26125	0·0425
190	25554	25513	25870	24960	0·1245
185	23703	Old, 23623 / New, 23763	24510	21660	0·4550
175	20259	Old, 20189 / New, 20271	22100	14060	1·6520
160	15900	15778	19090	− 20	4·710
150	13405	13262	17380	−10400	7·551

The three pressure-curves—which we shall name the "ordinary" vapour-pressure curve, the "superior" vapour-pressure curve, formed by the superior apices of the isothermal lines, and the "inferior" vapour-pressure curve, produced by the lower apices of the isothermal lines—must, it is evident, meet at the critical point; and on mapping them, it was found that this was the case. Points were chosen on these curves at equal intervals of temperature, and the constants for formulæ of the type $\log p = a + b\alpha^t$ were calculated for each. As the pressures on the inferior curve below a certain temperature were negative, it was found convenient to add 30,000 millim. to each, which was subsequently subtracted from the result. The constants for the curves are —

Superior curve :

$a = 3{\cdot}59797$; $\log b = \bar{1}{\cdot}8343195$; $\log \alpha = 0{\cdot}00257762$.

Ordinary vapour-pressure :

$a = 6{\cdot}72909$; $\log b = 0{\cdot}4027232$; $\log \alpha = \bar{1}{\cdot}99876897$

(b is here negative).

Inferior curve :

$a = 4{\cdot}867404$; $\log b = \bar{1}{\cdot}5913793$; $\log \alpha = \bar{1}{\cdot}98382413$

(b is again negative).

In each case $t = t°$ Cent. $- 160°$.

The results are given in the following Table :—

TABLE IX.

Temperature.	Ordinary vapour-pressures.		Superior curve-pressures.		Inferior curve-pressures.	
	Read.	Calculated.	Read.	Calculated.	Read.	Calculated.
°	millim.	millim.	millim.	millim.	millim.	millim.
150	13405	13084	17380	17437	−10400	−10185
160	15900	15900	19090	19090	− 20	− 100
175	20259	20259	22100	22100	+14060	+14060
185	23703	23678	24510	24549	21660	21704
190	25554	25556	25870	25935	24960	24900
192	26350	26341	26490	26523	26125	26065
193	.....	26739	.....	26826	.....	26624
193·83	.....	27075	.....	27077	.....	27077

With exception of the lowest temperature, the agreement between the read and the calculated pressures is close. The extrapolation amounts to only 3°·83. The agreement is close at 192°, and above that temperature the extrapolation is only 1°·83. It will be seen that at that temperature (193°·83) the pressures coincide. The *apparent* critical point was 193°·8.

Isochoric Lines.

Plate VI. represents the whole of the isochors which we have calculated between the volumes of 1·85 and 300 cub. centim. per gram.

If the gas followed Boyle's and Gay-Lussac's laws absolutely, under all conditions, the isochoric lines would all radiate from zero pressure, and would become more and more vertical as the volumes decreased; and the tangents of the angles formed by these lines with the horizontal line of zero pressure would be proportional to c in the equation $p=ct$, where c varies inversely as the volume. But our equation, $p=bt-a$, introduces another term, a, which is negative. These values of a are represented on the diagram by the extremities of the isochoric lines, where they cut the vertical line representing absolute zero of temperature. The tangents of the angles made by these lines with a horizontal line are proportional to the values of b in our equation.

On referring to Plate VI. it will be noticed that, beginning at the largest volume, two adjacent isochors cut each other at a point, as regards pressure and temperature, not far above

zero. With decreasing volumes the points of intersection of adjacent isochors occur at higher and rapidly increasing temperatures and slowly increasing pressures ; and this proceeds until the critical volume is reached. With still smaller volumes, however, the points of intersection of adjacent isochors occur at lower and decreasing temperatures and pressures ; the former decrease slowly, but the latter with great rapidity, and soon extend into the region of negative pressures.

It is evident from the diagram that each isochor between the largest and the critical volume is the tangent of a curve, representing the relations of pressure to temperature ; while the isochors below the critical volume are tangents to another curve, also exhibiting the like relations. Neither of these curves is identical with the vapour-pressure curve, which falls in the area between them.

It will be noticed that, in the area included between the line of zero pressure and these two curves, each isochoric line is cut by two others at every point along its whole length ; but outside this surface, and above the line of zero pressure, no two lines cut each other, and below the line of zero pressure each isochor is cut at each point by one other. The physical meaning of the fact that within the first-mentioned region three isochors intersect each other at one point is, that a gram of the substance may occupy three different volumes at the same temperature and pressure. Now, on referring to the diagram on Plate IV., representing the experimentally unrealizable portions of the isothermal curves, it is evident that on each isothermal line, at pressures limited by the superior or inferior apices of the isothermal, there are, corresponding to each pressure, three volumes. At any pressure above or below these pressures the isothermal line is cut only once, by a horizontal line of equal pressure ; so that, for each pressure, there is only one corresponding volume. At each apex a horizontal line of equal pressure cuts the isothermal line at one point, and is also a tangent to the apex. There are, therefore, two volumes corresponding to each of these pressures. Since no gas can be submitted to a negative pressure, those portions of an isothermal line representing the truly gaseous condition of matter never extend below the horizontal line of zero pressure ; only those portions of the isothermal

which proceed towards the inferior apex fall below this line. An isothermal line below zero pressure is therefore cut only twice by a line of equal pressure, and there are therefore two volumes corresponding to each pressure. At each inferior apex, however, the horizontal line is a tangent to the curve, and there is therefore only one volume corresponding to a given pressure.

On referring back to Plate VI., it will be seen that the pressures corresponding to the superior apices of each isothermal line, when mapped, produce the curve AC; and those corresponding to the inferior apices, the curve BC. The surface bounded by these curves and the line of zero pressure corresponds to portions of the isothermal lines, including pressures between the two apices, and each point in the surface is the locus of intersection of three isochoric lines. Below the line of zero pressure the isochoric lines corresponding to the gaseous state are absent; and hence each point is the locus of intersection of only two isochors. The isothermal lines above and below the limits of pressure given by the apices are cut only once by any line of equal pressure; hence the isochors outside the area ACD, and above the line of zero pressure, do not intersect. The apex C of the curvilateral triangle ACD is the point of highest temperature and pressure at which intersection can take place, and therefore represents the critical point; it is also the common point of intersection of the three pressure-temperature curves.

Referring now to Plate V., in which the isochoric lines in the neighbourhood of the critical point are shown on a larger scale, it will be seen that the isochoric lines above a volume not far removed from 4 cub. centim. per gram cut the ordinary vapour-pressure curve CE on one side, while those below the volume 3·75 evidently cut the vapour-pressure line on its other side. There must therefore be an isochoric line which does not cut the curve at all, but forms a tangent to its end-point. That isochor gives the critical volume. It may be determined by calculating the value of $\frac{dp}{dt}$ at the critical temperature. This value of $\frac{dp}{dt}$ is identical with the value of b in our equation $p=bt-a$ at the critical volume.

Until a mathematical expression is discovered, representing b as a function of volume, the only means at our disposal for ascertaining the true volume corresponding to b is by interpolation of the original curve by which the values of b were smoothed. The common point of intersection of the three pressure-temperature curves has been shown on p. 216 to lie at the temperature 193°·83. The value of $\frac{dp}{dt}$ on the vapour-pressure curve at this temperature, calculated by the formula of which the constants have already been given, is 405 millim., which is also the value of b at that temperature. The volume corresponding to this value is 4·06 cub. centim. per gram; and the specific gravity of ether at its critical point is therefore 0·2463.

Unfortunately, Dr. Andrews's measurements of the constants of carbon dioxide are not sufficiently numerous to warrant an attempt to obtain the critical temperature, pressure, and volume by this method. The critical volume of carbon dioxide is evidently less than 3 cub. centim. per gram; but the values of b below that quantity are unascertainable. It may be noticed that the curves below volume 3 are inserted in broken lines, showing a probable course; but no reading from them would be permissible.

The two liquids, ether and carbon dioxide, have no chemical analogy with one another; and we therefore feel justified in concluding that the law which is the subject of this paper is generally applicable to all stable substances. We have, however, other less complete data for methyl and ethyl alcohols, which, so far as they go, are confirmatory of the results described. We have also data available for the examination of acetic acid—a substance which differs from those mentioned, inasmuch as it undergoes dissociation when heated; and we hope shortly to be able to communicate an account of its behaviour.

Professor Fitzgerald, to whom we gave a short account of this law, has recently communicated to the Royal Society a paper in which its thermodynamical bearings are considered.

Bristol, 12th February, 1887.

XXX. *On the Determination of Coefficients of Mutual Induction by means of the Ballistic Galvanometer and Earth-Inductor.* *By* R. H. M. Bosanquet*.

My attention was drawn to this subject by the paper recently read before the Society by Prof. Foster†. I observed at once that the appliances which I am in the habit of using afford a very simple solution of the problem.

The ballistic galvanometer has a resistance less than 2 B.A. units, and about 500 turns. I have three earth induction-coils, all having a mean diameter of about half a metre, and the following constants. The resistances are approximate, as they vary so much with temperature. The wires are all cotton-covered copper.

Number of turns. N.	Total effective area. log NA.	Mean circumference.	Approximate resistance. B.A.	Wire. B.W.G.
42	4·97132	167·325	·6	16
250	5·74200	166·583	10·8	20
1000	6·36182	170·005	42	20

The circular channels are turned in wooden rings framed together in many pieces. They were turned in the Royal Society's lathe in my laboratory. They are mounted so as to turn on vertical axes through half a revolution, in doing which they are reversed with respect to the horizontal component of the earth's magnetism, and so experience an electrical impulse equal to 2 NAH.

If the resistance of the circuit be R, a transfer of electricity then takes place, such that

$$Q_0 = \frac{2\,NAH}{R}.$$

The various windings of the coils were measured with great care during the construction. The areas were calculated for each layer separately; and the above values of log NA are probably certain to the fourth place of decimals.

There are two tangent-galvanometers, both on Helmholtz's pattern, having mean circumferences of exactly 1 metre. The

* Read February 26, 1887.

† *Antè*, p. 137.

one has 2 coils, the other 18. The currents in them are measured by the formula

$$C = GH \tan \theta;$$

where log G has the values:—

No. of coils.	log G.
2	·24797
18	$\bar{1}$·29373

We can now proceed to the problem.

Two coils, P, S, are placed side by side, and a current passed through P; then, on making the contact, a ballistic galvanometer in the circuit of S is deflected. The earth induction-coil forms part of the circuit of S.

So far we have the equations

$$C = GH \tan \theta, \quad \ldots\ldots (1)$$

$$Q = \frac{MC}{R}, \quad \ldots\ldots (2)$$

with reading $= \alpha$ minutes of arc.

If we now give the earth induction-coil a half turn, we have

$$Q_0 = \frac{2NAH}{R}, \quad \ldots\ldots (3)$$

with reading $= \beta$ minutes of arc.

Whence
$$\frac{Q}{Q_0} = \frac{MG \tan \theta}{2NA}$$
$$= \frac{\alpha}{\beta}$$

by the principle of the ballistic galvanometer. And

$$M = \frac{\alpha}{\beta} \frac{2NA}{G \tan \theta}. \quad \ldots\ldots (4)$$

The 2 disappears from the numerator if the observation is made by reversal.

In order that H may be the same for the tangent-galvanometer and the earth induction-coil, it is necessary to remove the tangent-galvanometer and put the earth induction-coil in its place before making the observation β.

There are two classes of cases which cannot be treated by this most simple form of the method :—

(*a*) Where the quantity of electricity to be dealt with is too

large for the galvanometer, even with the coil of highest resistance. And

(*b*) where the resistance of one of the coils to be determined is so great that no earth induction-coil could throw a sufficient quantity of electricity through it.

In both these cases we have to introduce a large resistance into the circuit S, after calibrating the galvanometer without it. The value of the deflection is then altered in the ratio of the change of resistance.

Let R be the total increased resistance,

R_0 the resistance without the addition.

Then equations (2), (3), (4) become

$$Q = \frac{MC}{R}, \quad \ldots\ldots\ldots \quad (5)$$

$$Q_0 = \frac{2NAH}{R_0}, \quad \ldots\ldots\ldots \quad (6)$$

$$M = \frac{2\alpha RNA}{\beta R_0 G \tan\theta}. \quad \ldots\ldots \quad (7)$$

The 2 disappears, as before, when reversal is employed.

An example of case (*a*) is given later; where, in the second determination of my Gramme machine, a resistance of 1000 B.A. units was introduced into the secondary circuit, to moderate the deflection. The determination of the resistances, however, can hardly ever be accurate, as the copper earth induction-coil is liable to great alterations through change of temperature.

An example of case (*b*) would be afforded by the determination of such an induction-coil as the one by Apps, with high resistance S, mentioned in Prof. Foster's paper. In this case the galvanometer would be calibrated by the earth induction-coil, the high resistance-coil then introduced, and the known values of R, R_0 employed in formula (7).

The determinations I have made are mostly of the mutual coefficients of the coils of my earth-inductors, placed close together. I have also made a couple of determinations of the coefficient of field-magnets and armature in my A Gramme machine.

The determination of February 21 was rather rough; the coils were placed together in their frames, but they could not

be brought very near. The numbers are useful, as illustrating the increase of the discrepancy of Maxwell's formula, with the increase of b, the distance of the central planes. In fact, unless b is small compared with the diameters, the formula is not properly applicable. Here it is about half either of the radii, and calculated and observed values are nearly as 4 : 5.

On February 22 the coils were dismounted and brought as close together as possible, b being now between a quarter and a third of either of the radii. The calculated and observed values are nearly as 5 : 6. In order to see what part of the error was due to the dimensions of the section of the coils, the coefficient was then calculated by the more accurate formula referred to below; but the improvement is very slight. It appears that this formula does not in any way cure that defect, which arises from the distance of the central planes. So that in coils similar to mine we cannot expect to get nearer by calculation than the number last referred to, which is to the observed number nearly as 6 : 7.

On February 23 experiments were arranged with a view to test the consistency of the method, the coils examined being made primary and secondary alternately. Though the mode of observation does not admit of great accuracy, it appears that there is a systematic difference between the results of the two arrangements, amounting to about $1\frac{1}{2}$ per cent. This I have not been able to explain. The coefficients of self-induction of the coils should have no influence; but that of the 250 will be very much greater than that of the 42, and this is the only source of error that I can suggest. The number calculated by Maxwell's original formula is to the mean of observation nearly as 6 : 7.

In all the experiments difficulty was experienced in consequence of the continual fall of the battery-current. Without care in charging, and freshly charged cells, the experiments could hardly be made, as the fall of the current affects the galvanometer in the intervals.

Rowland's method of control, with a magnet and coil, was employed.

The determinations of the coefficient of the Gramme machine were very rough. In fact the fundamental equation (2) is not really applicable to the dynamo at all, except perhaps in the upper part of its range, where the current is, say, 10

amperes or more. For small currents do not do more than shake the subpermanent magnetism, which is considerable compared with the magnetism due to small currents. And in motion the machine makes use of this subpermanent magnetism, although it does not enter into the electrical coefficient of mutual induction.

The observations of Feb. 24 were very irregular; it appeared as if the current sometimes shook the subpermanent magnetism and sometimes not. Still a mean was fairly deducible.

On Feb. 25 a stronger current was employed, and the observations were fairly regular; but this current, though it moves the subpermanent magnetism, is quite insufficient to reverse it as the reversal of a large current does; so that even here we do not get a representation of the whole effect.

In the higher part of the range of the machine, say for unit-current (10 amperes), we can assume, without serious error, that the fundamental equation (2) holds. That being so, we can deduce the value of M from the resistance and number of revolutions with which a stable current is produced.

It will be convenient to assume that all the coils of the armature are gathered up into two coils at right angles. Then, if one be parallel and the other perpendicular to the axis of the magnetic field, the determination of the coefficient of induction at rest affects only one of the coils, or half the armature.

Then, as to one of these coils, or half the armature:—In a half revolution the same effect is produced as if the exciting current were reversed, and a quantity of electricity passes,

$$=\frac{2MC}{R}.$$

In a whole revolution $\frac{4MC}{R}$ passes.

The same applies to the other half of the armature, so that altogether $\frac{8MC}{R}$ passes in each revolution.

If there be n revolutions per second, the quantity passing in n revolutions = the current numerically, and this divides

out, so that

$$M=\frac{R}{8n}.$$

In an old paper of mine on Practical Electricity (Phil. Mag. xiv. p. 246) I find data from which, by interpolation, I obtain the corresponding values for this machine :—

n' per minute.	R.	C.
840	$10^9\times5{\cdot}55$	1=10 amperes.

Whence $M=10^7\times4{\cdot}95$.

The assumption made above is rough, but we cannot specialize further unless the distribution of the field is known. The two coils may then be supposed to be in any required proportion instead of being equal.

Regarding the coefficient as the product of the number of windings of armature and magnets, and of the magnetic induction common to both, due to unit-current, we can find the common magnetic induction in this case. There are 1700 turns in the armature, half of which count, and 234 turns on each of the four magnets, two of which receive each part of the common induction. Whence the common magnetic induction is

$$\frac{10^7\times4{\cdot}95}{468\times850}=125.$$

This seems very small. But the dimensions of the armature are such that it can have hardly any core.

But I must not now pursue the subject of the dynamo.

The method of the earth induction-coil and ballistic galvanometer is susceptible of numerous applications. I have in my mind the direct determination of the capacity of a condenser, and a method following the lines of that used by Weber, for the determination of resistance. He employed earth induction-coils on this principle, though I do not know how they were arranged in detail. I cannot conceive how a tangent galvanometer could be employed for the ballistic work, as seems to be contemplated in Maxwell's account. I should determine the constant of the ballistic galvanometer for the purpose by a shunt comparison with a tangent galvanometer. I believe Mr. Glazebrook has done something of this kind in another case. In fact I have been through the work of such a method; but where there is a shunt to be verified, in this case 1 : 10,000, as well as other determinations of resistances, the errors of temperature are so trouble-

some to deal with, that in the present state of the subject I doubt whether it is worth while to spend much labour on it.

The numerical results of the experiments referred to are appended.

Coefficients of Induction of 250 *and* 100 *coils.*

Feb. 21. Distance between central planes, $b=13{\cdot}97$ centims.
β by half turn of 1000 coil.

Arrangement.	α.	β.	θ.	Galvanometer.		M.
250 P 1000 S	152′·5	164′·5	15° 45′	18	make and break	$10^7\times 7{\cdot}69$

*Calculation by Maxwell's original formula . . . $10^7\times 6{\cdot}16$

Feb. 22. Distance between central planes, $b=7{\cdot}925$ centim.
β by half turn of 1000 coil.

Arrangement.	α.	β.	θ.	Galvanometer.		M.
250 P 1000 S	203′·7	170′·	12° 33′	18	make and break	$10^8\times 1{\cdot}25$

*Calculation by Maxwell's original formula . . . $10^8\times 1{\cdot}07$
*Calculation by formula for dimensions of section of coils $10^8\times 1{\cdot}10$

Data for this last formula :—

$n=1000$ $\quad n'=250$
$a=27{\cdot}056$ $\quad a'=26{\cdot}512$
$b=7{\cdot}925$
$2h=2{\cdot}18$ $\quad 2h'=\cdot 562$
$2k=5{\cdot}45$ $\quad 2k'=5{\cdot}77$

Coefficients of Induction of 42 *and* 250 *coils.*

Distance between central planes, $b=6{\cdot}35$ centim.
β by half turn of 1000 coil, in S, throughout.

Date.	Set.	Arrangement.	α.	β.	θ.	Galvanometer.	Make & break or reversal.	M.
Feb. 23.	1	42 P 250 S	202·2	142′	31° 56′	2	m. b.	$10^6\times 5{\cdot}9390$
	2	250 P 42 S	88·2	173	44 20	18	rev.	$\times 6{\cdot}1075$
Feb. 24.	1	.....	86·5	175	43 55·5	...	...	$\times 6{\cdot}0030$
	2	.....	85·2	175	43 5	...	...	$\times 6{\cdot}0893$
	3	42 P 250 S	190·2	144·7	29 43	2	m. b.	$\times 5{\cdot}9862$

Calculation by Maxwell's original formula . $10^6\times 5{\cdot}275$

* New edition of Maxwell's 'Electricity and Magnetism,' vol. ii. pp. 314, 320.

Comparison of arrangements :—

		42 P 250 S			250 P 42 S	
Feb. 23.	1	$10^6 \times 5{\cdot}9390$	Feb. 23.	2	$10^6 \times 6{\cdot}1075$
„ 24.	3	$\times 5{\cdot}9862$	„ 24.	1	$\times 6{\cdot}0030$
			„ „	2	$\times 6{\cdot}0893$
	Mean	$\times 5{\cdot}9626$		Mean	$\times 6{\cdot}0666$

Mean of both arrangements $10^6 \times 6{\cdot}0146$.

Determinations of Coefficient of Induction of Dynamo Field-magnets and Armature. (A Gramme.)

Feb. 24. P field-magnets, with two bichromates and 10 B.A. units.

S armature, ballistic galvanometer, and 1000 coil.

By reversal.

	α.	β.	θ.	Galvanometer.	Amperes.	M.
Mean value . .	146′	179′	27° 11′	18	·181	$10^7 \times 1{\cdot}858$
Highest value .	176	...	.....	...	...	$\times 2{\cdot}239$
Lowest „ .	113	..	.....	...	...	$\times 1{\cdot}438$

Feb. 25. P field-magnets, with two bichromates.

S armature, ballistic galvanometer, and 1000 coil, with and without 1000 B.A. units, so that

$R = 1045$, $R_0 = 45$.

By reversal.

α.	β.	θ.	Galvanometer.	Amperes.	M.
179′·5	179	39° 24′	2	2·54	$10^7 \times 3{\cdot}685$
		By formula $M = \frac{R}{8n}$		10·0	$10^7 \times 4{\cdot}95$

Addendum.

March 5. Supplementary Notes.

1. Calculation of coefficients by the elliptic integral table in the new edition of Maxwell, combined with the formula for approximation to the effect of the sections of the coils.

I have now calculated the experiments of Feb. 22 and Feb. 23–24 by this method, which is the most complete that exists, short of calculating all the single combinations of circles in the two coils. The accordance with experiment is somewhat better, but still far from close.

	Observed.	Calculated.
Feb. 22.	$10^8 \times 1{\cdot}259$	$10^8 \times 1{\cdot}176$
„ 23–24.	$10^6 \times 6{\cdot}015$	$10^6 \times 5{\cdot}585$

2. Simple formula for approximate calculation of the coefficient.

Assume that the field in a due to A is everywhere the same as at the centre of a. Then the total lines of force for unit current are

$$\pi a^2 \times \frac{2\pi A^2}{(A^2+b^2)^{\frac{3}{2}}},$$

where b is the distance between the planes of the circles, or, if $\frac{A}{b} = \tan\theta$, this becomes

$$\frac{2\pi^2 a^2 \sin^3\theta}{A},$$

which is simpler to calculate than Maxwell's approximate formula.

The following is the comparison of this formula with observation.

	Observed.	Calculated.
Feb. 21.	$10^7 \times 7{\cdot}690$	$10^7 \times 8{\cdot}991$
„ 22.	$10^8 \times 1{\cdot}259$	$10^8 \times 1{\cdot}133$
„ 23–24.	$10^6 \times 6{\cdot}015$	$10^6 \times 5{\cdot}036$

3. Course of values of the coefficient of field-magnets and armature of a dynamo.

The numbers stated in the paper may possibly be misleading, as it is not sufficiently explained that the number determined from the motion of the machine, $\left(\frac{R}{8n}\right)$, is not of the same nature as the two results of determinations made at rest, which precede.

The number determined from $\left(\frac{R}{8n}\right)$ is necessarily infinite when the current is evanescent, if there is any retention of magnetism in the machine, and diminishes continually as the current increases. The following corresponding values of this coefficient and current are given by the data referred to in the paper.

Coefficient.	Current in amperes.
$10^7 \times 7{\cdot}054$	3·8
$\times 6{\cdot}250$	5·5
$\times 5{\cdot}600$	7·6
$\times 4{\cdot}732$	10·5

XXXI. *On Delicate Thermometers.* *By* Spencer Umfreville Pickering, *M.A., Professor of Chemistry at Bedford College**.

Some months ago I had the honour of bringing before the notice of this Society (*antè*, p. 8) the fact that with very delicate thermometers the temperature registered was never exactly the same when the column had risen to the point of rest as when it had fallen to it. The investigation was made by placing the instrument in a large calorimeter of water, removing the bulb at intervals, cooling or heating it slightly, and then replacing it and observing the reading. About eight separate observations were made in order to determine the difference between the falling and rising readings at any particular point on the stem; and throughout the observations the instrument was tapped continuously on the upper end † to overcome the inertia of the mercury in the tube. The difference which was noticed was explained at the time by the bulb not being of precisely the same shape while the mercury was being forced upwards through the fine tube as when it was being dragged downwards; nevertheless there were several points which rendered such an explanation not altogether satisfactory. One out of the two instruments examined (No. 62839) gave differences proportional to the height of the column in the tube; in the other (No. 63616) no such proportionality was noticed: moreover, the instrument which showed this defect to the greatest extent (63616) was the one which had the smallest and strongest bulb. I was subsequently indebted to Lord Rayleigh for a suggestion that these differences should be attributed to the capillarity of the tubes acting on the expansibility of the bulbs, and not to the action of the bulbs only, and thus the differences observed between the behaviour of the two instruments might easily be accounted for by slight differences in the shape or size of the bore at various points. In order to test the validity of such an explanation, which at first sight appeared highly probable, other experiments were performed

* Read April 23, 1887.

† I now employ a clockwork tapping apparatus for this purpose.

with these instruments, taking care to make all the observations at exactly the same temperature, removing some of the mercury into the upper chamber, so as to make the different points on the scale correspond to the same Centigrade temperature. The somewhat bulky details of the experiments may here be omitted, and the general results only given. Table I. contains these results; those which were obtained originally, and have been given in the previous

TABLE I.

Thermometer No. '39.

Scale-Reading	465	457*	270*	162	142* mm.
Difference in falling and rising readings.	0·5	·46*	·27*	·17	·15* mm.
Relative capacity of tube at the point taken	46·1	46·1	45	45·6	45·6

Thermometer No. '616.

Scale-Reading	562	458	458*	289	154	142	140*
Difference in falling and rising readings.	0·73	·42	·96*	78	·67	·62	·91*
Relative capacity of tube at the point taken	40·3	40	40	39·3	40·1	40·1	40·1

communication, are marked by an asterisk; and any small discrepancies observed between them and the fresh experiments may be attributed to differences in the actual temperature of the observations, the recent experiments only being strictly comparable with each other, having all been performed at 10°·5 C. The readings and the differences are expressed here in mm. of the column, instead of cm. (or scale-degrees) as in the former communication.

To whichever thermometer we confine our attention, it is perfectly obvious that the magnitude of the differences bears no relation to the size of the bore. Taking the original experiments with '39, the difference as judged by the size of the bore should be least at 457, and greatest at 270, and of intermediate value at 142 mm.; whereas they are found to be greatest at 457 and least at 142 mm. Again, with No. '616, the differences should be greatest at 289, and least at 562 mm.; whereas they are practically equal at these two points, and greater (taking the later experiments only) than at any other points. It may be objected, however, that the capacity of the tube at these points, as given by the length

of a comparatively long (2 cm.) thread of mercury in that position, affords but a very rough, and perhaps quite erroneous, measure of the diameter of the tube at the particular point in question. As a crucial test, therefore, it was determined to alter the bulb of one of the instruments. This was done with Thermometer '39. The capacity of the bulb was reduced to half its former value, so as to hold 18 instead of 36 grams of mercury; and a determination of the coefficients of expansion of the two bulbs under pressure (given in column 6 of Table II.) shows that what may be termed the apparent expansion, or the effect of pressure on the height of the mercury in the tube, was thereby reduced to nearly half its former value, from ·042 to ·025 mm. per mm. of mercury-pressure, so that the differences in the readings with falling and rising columns should have been diminished in that proportion, if it depended on the expansion of the bulb by pressure; but on examination it was found that this difference was even greater now than it had been previously—at 270 mm. it amounted to ·59 instead of ·27 mm., and at 457 mm. it was ·39 instead of ·46 mm. (col. 7 of Table II.). Another thermometer (No. '83) was then examined in a similar manner. Originally it exhibited no difference whatever in the falling and rising readings; but when the bulb was altered, without increasing the coefficient of apparent expansion to any considerable extent (from ·011 to ·015), a small though unmistakable difference in the readings was observed (No. '83 B in the table). This second bulb was then removed, and a third and smaller one substituted for it (No. '83 C), by which the coefficient of apparent expansion was reduced by one half its former value; but, instead of the differences being diminished, they were actually increased, although no accurate measure could be made of them, for the column of mercury in the tube kept breaking off when the instrument was tapped. It was evident, therefore, that the cause of these differences in the readings did not lie in the bulbs of the instruments, but in the stems, that each time the instrument was opened and air admitted into the stem, the defect was increased, till the tube eventually became entirely ruined. The moisture and gases present in the air, no doubt, affect the glass and adhere so strongly to it that the heating to which the stem is sub-

Table II.

Instrument.	Mercury in bulb.	Relative capacity of tube.	Value of 1 mm. of column.	Coeff. of expansion of bulb per mm. mercury-pressure.	Alteration in height of column per mm. mercury pressure.	Difference of readings with falling and rising column.	Description of bulb.
					mm.		
'616.	30·5 grams	40	0·0064 C.	0·000,000,034,8	0·0303	0·42 to 0·78 mm.	Double, glass cylinder.
'39.	36·2 grams	45	0·006 C.	0·000,000,045,8	0·0424	0·15 to 0·45 mm.	Single, glass cylinder.
'39 B.	18·1 grams	45	0·012 C.	0·000,000,054,3	0·0252	0·39 to 0·59 mm.	Single, blown.
'83.	15·6 grams	100	0·038 C.	0·000,000,064,6	0·0114	Nil.	Single, blown.
'83 B.	28·5 grams	100	0·027 C.	0·000,000,041,5	0·0147	0·04 to 0·32 mm.	Single, glass cylinder.
'83 C.	19·4 grams	100	0·0304 C.	0·000,000,03	0·0075	Large.	Single, glass cylinder.
'08.	38·8 grams	76	0·0116 C.	0·000,000,030,1	0·0145	Nil.	Double, glass cylinder.
'61.	46·0 grams	72	0·0097 C.	0·000,000,033,2	0·0191	Nil.	Double, glass cylinder.
'16.	15·4 grams	100	0·0385 C.	0·000,000,035,6	0·0062	Nil.	Single, glass cylinder.

jected is quite incapable of removing it, and the interior of the tube remains coated with an elastic covering which destroys the working capabilities of the instrument. The researches of Bottomley (Proc. Roy. Soc. xxxviii. p. 158) and others on the absorption of air, and especially carbon dioxide, by glass, prove, in a striking manner, the extent to which such absorption occurs, and the persistence with which the absorbed gases are retained.

Both the delicate instruments ('39 and '616), in which the difference of the readings was first observed, had had small temporary bulbs attached for the purpose of calibration; and it therefore seemed probable that the second opening of the tube may have been the sole cause of the defects which they exhibited. To settle this question, and to ascertain whether it was possible to make instruments of such delicacy entirely free from this defect, two other thermometers were manufactured, Nos '08 and '61. The delicacy of these was somewhat less than in the former instruments, owing to the impossibility of procuring sufficiently fine tubes; the size of the bulbs, however, was increased, that of '61 containing as much as 46 grams of mercury. An estimation-figure, ·05 mm., represented about 0·0005°C. On examining these instruments, in the same manner as previously, it was found that they worked perfectly, the mercury registering exactly the same temperature whether the column had risen or fallen to the point of rest, equally satisfactory results being obtained whatever portion of the stem was examined. Instruments of this excessive delicacy are therefore perfectly workable; it is, however, only by observing the utmost precaution in making them that success can be obtained. The tube must on no account be opened till the last minute, when the bulb is finished and ready to be attached without a moment's delay; the bulb, as soon as it is attached, must be warmed so as to fill the tube with mercury and prevent the access of air through the upper end. If any failure occurs in the attachment of the bulb at the first trial, the stem must be rejected; a second attempt would be attended with the same results as putting on a second bulb after the instrument had been made up. When once the stem is filled with mercury, the tube may apparently be opened several times at the top without damage being done,

and the bulb itself may be made 24 hours before it is attache to the stem without being injured by exposure to air for th time.

It is a common practice of thermometer-makers to examin the bore of a tube before it is made into a thermometer b passing a thread of mercury along it, and often, indeed, th stems are divided and fully calibrated before the bulb attached and the tube closed. From what has been ascertaine as to the effect of the air on the interior of the tube, it is obviou that a tube which has been treated in such a manner will b utterly useless for any really delicate instrument.

XXXII. *On the Effect of Pressure on Thermometer-bulbs, and on some Sources of Error in Thermometers.* *By* Spencer Umfreville Pickering, *M.A., Professor of Chemistry at Bedford College**.

The great difficulty which exists in obtaining exact concordance between two thermometers throughout a considerable range of their scales must have been experienced by all who have had occasion to require such concordance. In the course of a series of experiments, in which the temperature-disturbance in a calorimeter was measured simultaneously with two instruments, I was much struck by the appearance of a certain amount of regularity in the difference in the results yielded by the two instruments, which, according to direct comparisons with each other through longer ranges of temperature, should have been absolutely concordant. The instruments being open in the scale and having large bulbs, I was led to seek for an explanation of these discrepancies in irregularities in the expansion of the bulbs under the pressure of the column of mercury in the tube.

The effect of pressure on a thermometer-bulb has been investigated by Egen (Pogg. *Ann.* xi. p. 283), and by Mills (Roy. Soc. Edin. xxix. p. 285), with the general result of showing that the expansion experienced is directly proportional to the pressure. But although the pressures employed

* Read April 23, 1887.

in Mills's experiments were considerable (ranging up to 134 atmospheres), the thermometers which he examined were not of the most delicate character, and the coefficient of expansion was but small in comparison with that possessed by most calorimetric instruments. In the present experiments the bulb of the thermometer was enclosed in a small thin brass cylinder, which was connected with a pump, by means of which a vacuum, or pressure up to two atmospheres, might be produced. The case enclosing the bulb was previously filled with melting ice and placed in a large vessel of the same, the zero point of the thermometer having been previously adjusted so as to be slightly above the level of the ice.

Three instruments were investigated in this manner, Nos. 62839, 63616, and 65108, of which the details of construction are given in the table ii. of the preceding communication (see also *antè*, pp. 9, 19). About twenty observations at different pressures were made in each case, and the results are given in Table I., where are entered the observed pressures and readings in millim., and also the readings, calculated on the assumption that the alteration in height of the column is directly proportional to the pressure; these calculated values being deduced from the average points given at the foot of the table. With the last-mentioned instrument (65108) the effect of pressure on the bulb would appear to cause a very regular expansion; there are only three observations which differ from the calculated values by more than 0·1 millim., while the average difference amounts to about 0·05 millim. only. With the other two instruments the observations at the higher pressures cannot be much relied on, since the pressures were ascertained by means of a Bourdon's gauge, instead of a column of mercury as in the other cases; and this gauge was afterwards found to be untrustworthy. Omitting these observations, we find that, in case of No. 63616, where the coefficient of apparent expansion of the bulb was twice as great as that of 65108, the error in the readings is considerably larger, amounting to as much as 0·12 millim. on the average, while the grouping together of the positive and negative differences is well marked. It is only with No. 62839, however, which possessed a still greater coefficient of expansion, that the differences become so large

TABLE I.—Effect of Pressure on the Bulbs of Thermometers.

Thermometer 62839.			
External pressure in mm. of mercury.	Observed reading.	Calculated reading.	Difference.
	mm.	mm.	mm.
1986	97·80	99·40	−1·60
1718	86·60	89·00	−2·40
1438	75·35	77·15	−1·80
1254	68·70	69·35	−0·65
1093	62·65	62·50	+0·15
928	56·25	55·50	+0·75
851	52·85	52·25	+0·60
723	48·00	46·80	+1·20
665	45·55	44·35	+1·20
596	42·55	41·45	+1·10
516	38·15	38·05	+0·10
416	33·80	33·80	0
316	29·45	29·55	−0·10
216	25·50	25·30	+0·20
173	23·55	23·50	+0·05
118	21·20	21·15	+0·05
82	19·55	19·60	−0·05
47	18·15	18·15	0
11	16·55	16·60	−0·05
−22·5	15·10	15·20	−0·10
−55	14·00	13·80	+0·20

Average points, 92·5 mm. at 1800 P. and 16·68 mm. at 12·5 P.

Thermometer 63616.			
External pressure in mm. of mercury.	Observed reading.	Calculated reading.	Difference.
	mm.	mm.	mm.
1960	94·00	94·80	−0·80
1738	87·60	88·10	−0·50
1433	78·75	78·80	−0·05
1180	71·45	71·05	+0·40
1009	66·05	66·00	+0·05
874	61·65	61·85	−0·20
724	57·25	57·30	−0·05
667	55·85	55·60	+0·25
596	53·60	53·45	+0·15
515	51·25	51·00	+0·25
414	48·05	47·90	+0·15
320	44·95	45·05	−0·10
216	41·90	42·00	−0·10
163	40·25	40·30	−0·05
117	38·65	38·90	−0·25
66	37·40	37·35	+0·05
16	35·90	35·85	+0·05
−25	34·60	34·60	0
−56	33·80	33·65	+0·15

Average points, 67·03 mm. at 1044 P, and 36·07 mm. at 23·6 P.

Thermometer 65108.			
External pressure in mm. of mercury.	Observed reading.	Calculated reading.	Difference.
	mm.	mm.	mm.
1771	43·00	42·95	+0·05
1738	42·45	42·50	−0·05
1660	41·35	41·35	0
1597	40·40	40·45	−0·05
1521·5	39·35	39·35	0
1435	38·15	38·10	+0·05
1328	36·55	36·45	+0·10
1228	35·15	35·10	+0·05
1131·5	33·65	33·70	−0·05
1052	32·25	32·55	−0·30
932	30·85	30·80	+0·05
832·5	29·35	29·35	0
734·5	28·00	28·00	0
634	26·40	26·50	−0·10
519·5	24·80	24·85	−0·05
418·5	23·45	23·40	+0·05
323	22·15	22·00	+0·15
221·5	20·60	20·50	+0·10
125·5	19·15	19·15	0
26	17·75	17·70	+0·05
− 14·5	16·90	17·10	−0·20
− 70·5	16·40	16·30	+0·10
−109	15·75	15·75	0

Average points, 41·32 mm. at 1657·5 P, and

as to render it quite impossible to attribute them to mere experimental error. As the pressure is increased above 500 millim., the bulb begins to contract far more than it should, causing a difference of as much as 1·2 millim. between the observed and calculated readings; it then contracts less, crosses the line representing the calculated values and for pressures from 1100 up to 1986 millim. (if these higher results may be trusted) does not contract as much as it should do*. The action of the bulb under pressure is evidently not regular.

As the differences in the calorimetric results above mentioned were observed with the thermometers '616 and '08, which behaved normally, or nearly so, under pressure, as well as with other instruments with bulbs of considerably smaller expansibility, it is impossible to attribute these differences to the cause suggested; at the same time, however, the present investigation leads to results of considerable practical importance. It is evident that where the coefficient of expansion of the bulb is large, as with '39, irregularities in expansion sufficient to introduce considerable errors may occur; the bulb, when subjected to pressure, would appear to behave in a manner analogous to that often noticed with thin tin plate vessels, where a small addition or removal of pressure will cause a sudden and considerable alteration in the form of the vessel. It would certainly be advisable, in the case of any thermometer required for very delicate work, to examine it under pressure to ascertain whether its action be uniform or not. In order to reduce the chances of irregular action, it is necessary to render the bulb as inexpansible as possible. From the fact that the thermometer 63616 showed a slight amount of irregularity in its action, we may place the limits of expansibility desirable in a very fine instrument at a number between that of this instrument and that of 65108; the coefficient of expansion should not exceed 0·000,000,03, or the apparent expansion 0·02 millim. of the mercurial column per millim. of pressure.

* Care was of course taken that the thermometer should not be read till the column had attained a position of equilibrium. The top of the instrument was tapped throughout the experiments by means of the tapping apparatus, and the column read at intervals of one minute till it was found to be perfectly stationary.

An examination of table ii. of the preceding communication will show at once the increase of rigidity obtained by having the bulb made out of a glass cylinder instead of being blown before the lamp; the instruments '83 and '39 B are the only ones mentioned in this table which had blown bulbs, and the coefficients of expansion in their cases are higher than in any other case, although their bulbs were only $\frac{1}{2}$ and $\frac{1}{3}$ as big as those of most of the other instruments. Again, the thermometers '83 and '16 were identical in all respects except as regards their bulbs, and here the blown bulb ('83) possesses only half the strength of that made out of cylinder.

Although it seems probable, *prima facie*, that a blown bulb, however well constructed, would not be so uniform as a cylinder bulb, these facts of course do not prove that such is necessarily the case, as the thickness of the walls of the bulbs was not known; but it does prove that, by ordering a thermometer with a cylinder bulb, we should in all probability get an instrument possessing nearly twice the strength of one with a blown bulb. A further very considerable addition of strength may be gained by having the bulb made double instead of single. The instruments Nos. '616, '08, and '61 possessed double bulbs made out of glass cylinder; and a comparison of them with '83 B and '39, which had single bulbs made out of cylinder also, will show the advantages of the double bulb. For thermometers to be used in a liquid which is stirred in a thoroughly efficient manner, the thickness of the walls of the tube forming the bulb may be very considerable. The instruments Nos. '08 and '61, which contain between 40 and 50 grams of mercury, will take the temperature of the liquid in the calorimeter in about 5 seconds, although their bulbs are 0·75 millim. thick in the walls.

Although the effect of different pressures on the bulb did not appear to produce irregularities such as would account for all the difficulties experienced in getting two instruments to correspond perfectly throughout their scales, it was thought possible that the expansibility of the bulbs under pressure might be influenced to a considerable extent by the temperature of the experiment, and that this might produce discordance between two instruments which had been compared with the same standard but at different temperatures. To

investigate this, the behaviour of No. '08 under pressure was examined at 12° C. As it was impossible to keep the temperature of the bath absolutely constant throughout the series of experiments, the reading at any pressure P_1 was compared with that at P by taking the mean of two observations at P, made immediately before and immediately after the observation at P_1. In the following Table the value of the constant

Thermometer '08. Under pressure at 12°.

P_1-P.	$\frac{R_1-R}{P_1-P}$.
millim.	
−500	0·01445
−400	0·01445
−300	0·01434
−200	0·01380
+200	0·01470
,,	0·01469
+300	0·01438
,,	0·01462
+400	0·01447
+500	0·01455
+600	0·01457
Mean	0·01453

obtained from $\frac{R_1-R}{P_1-P}$ (R_1 and R being the readings at P_1 and P respectively) is given in the second column, while P_1-P is given in the first column. P was in all cases the atmospheric pressure. The mean value of the constant (the coefficient of apparent expansion) is 0·0145, a number absolutely identical with that found at 0° (table ii., preceding paper), which shows that small alterations of temperature, such as would occur in standardizing delicate calorimetric thermometers, produce no appreciable alteration in the rigidity of the bulbs. The causes producing non-concordance of thermometers under certain circumstances still remains to be discovered*.

I may mention one source of error in thermometric work

* For an unexplained instance of non-accordance of results obtained with different thermometers, and also with different portions of the stem of the same thermometer, see Chem. Soc. Trans. 1887, pp. 304, 322.

which attains considerable dimensions when dealing with tub of very fine bore. These tubes, even when of the most perfe description obtainable, generally possess a few points at whic the mercury column experiences a difficulty in passing: th mercury, when it has reached such a point, sticks there a appreciable time, and then passes it suddenly with a jerk sometimes, even, the mercury sticks so persistently that th column will separate sooner than pass it. These points do no indicate any contraction which is sufficient to affect the resul of calibration, and are probably due to some difference in th nature of the glass, for they may be developed by heatin the thermometer-tube externally with a very small flame u to about 400°. The error of taking a reading while th mercury is sticking at such a point may, I estimate, amoun sometimes to as much as 0·5 millim. All delicate thermo meters should be carefully examined in order to ascertain th position of such points, and they should be avoided, if possible in any work with the instrument.

In any thermochemical work in which the effect of tempe rature on a given reaction is being studied, many of th sources of error inherent in the use of thermometers may b avoided by using the same portion of the stem of the instru ment, whatever the actual temperature may be. To effec this, the zero-point is altered in each experiment by removing some of the mercury into the upper chamber of the thermo meter. Formerly I removed the requisite amount of mercury by the application of a very small flame to a point just below this chamber; but I now adopt a method which is much safer, more expeditious, and equally exact. A fine tube somewhat wider than the stem of the thermometer, is affixed to the upper end of the stem, and in this tube there is a smal contraction or "knife-edge," sufficiently wide to permit of the mercury passing it either upwards or downwards, but yet so narrow that a slight swing of the instrument will cause the column of mercury to break off at it, that portion of the mercury which is above the knife-edge passing up into the upper chamber. By this means any point on the stem may be adjusted with ease to within 0°·02 of any given tem- perature.

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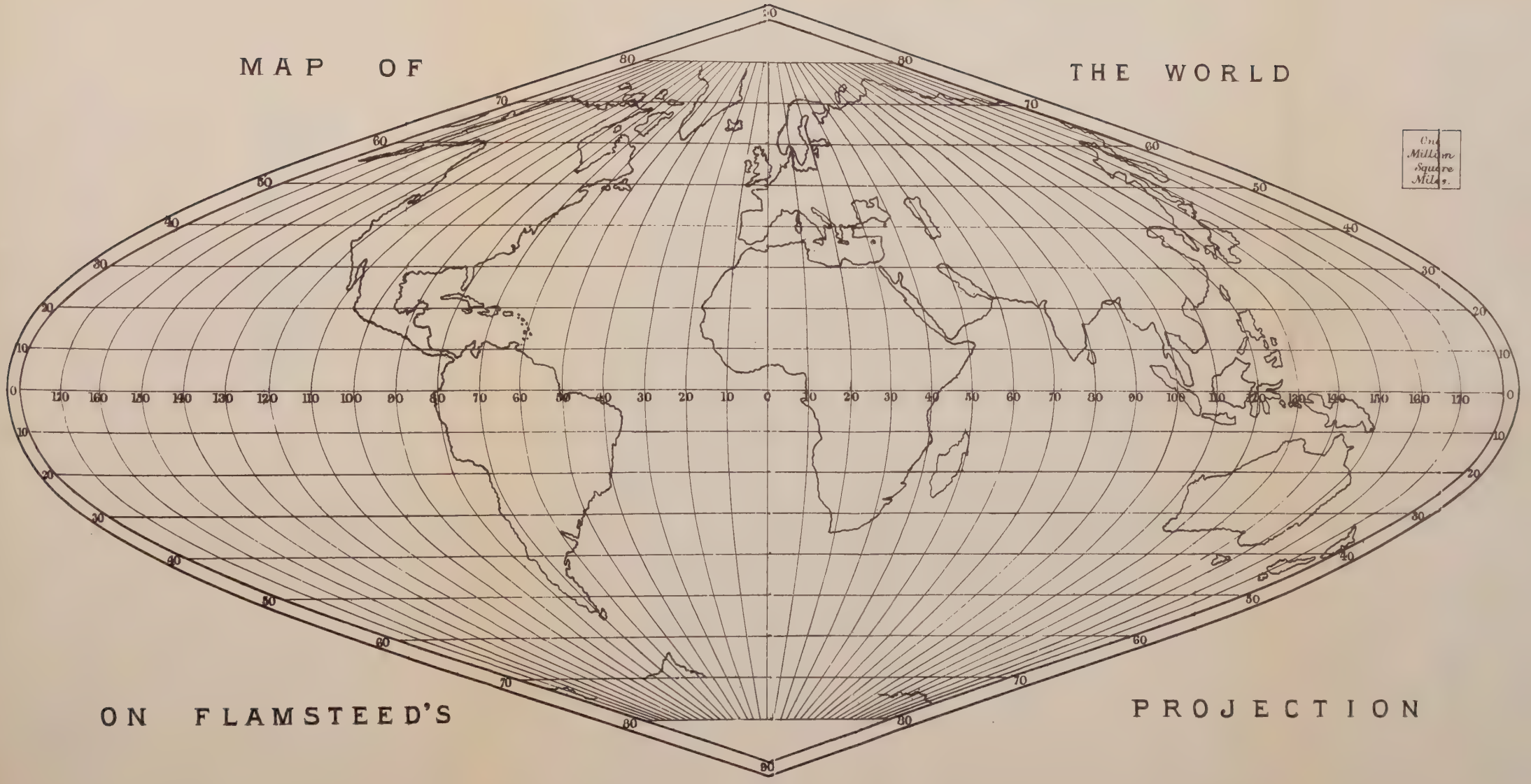
Proc. Phys Soc vol. 8. 1884.

MAP OF THE WORLD ON FLAMSTEED'S PROJECTION



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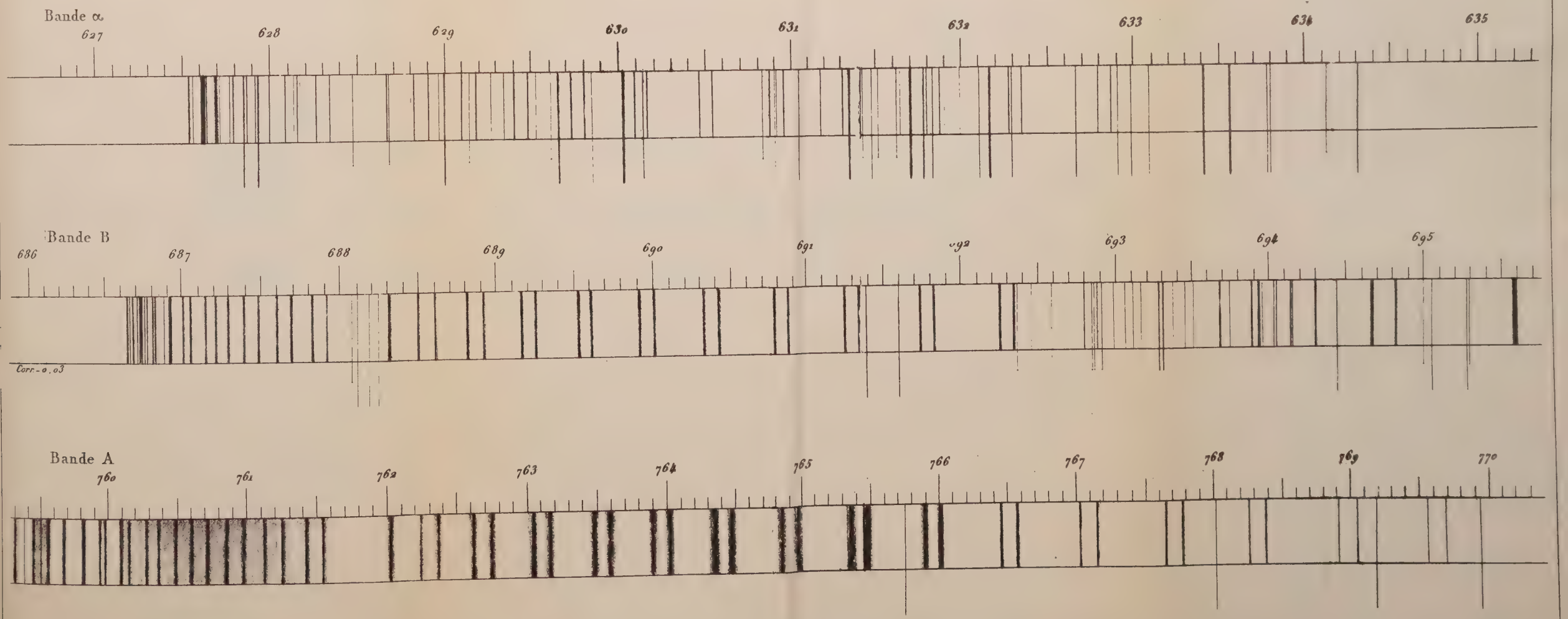
Etude des bandes telluriques α, B et A du spectre solaire
par M. A. Cornu.



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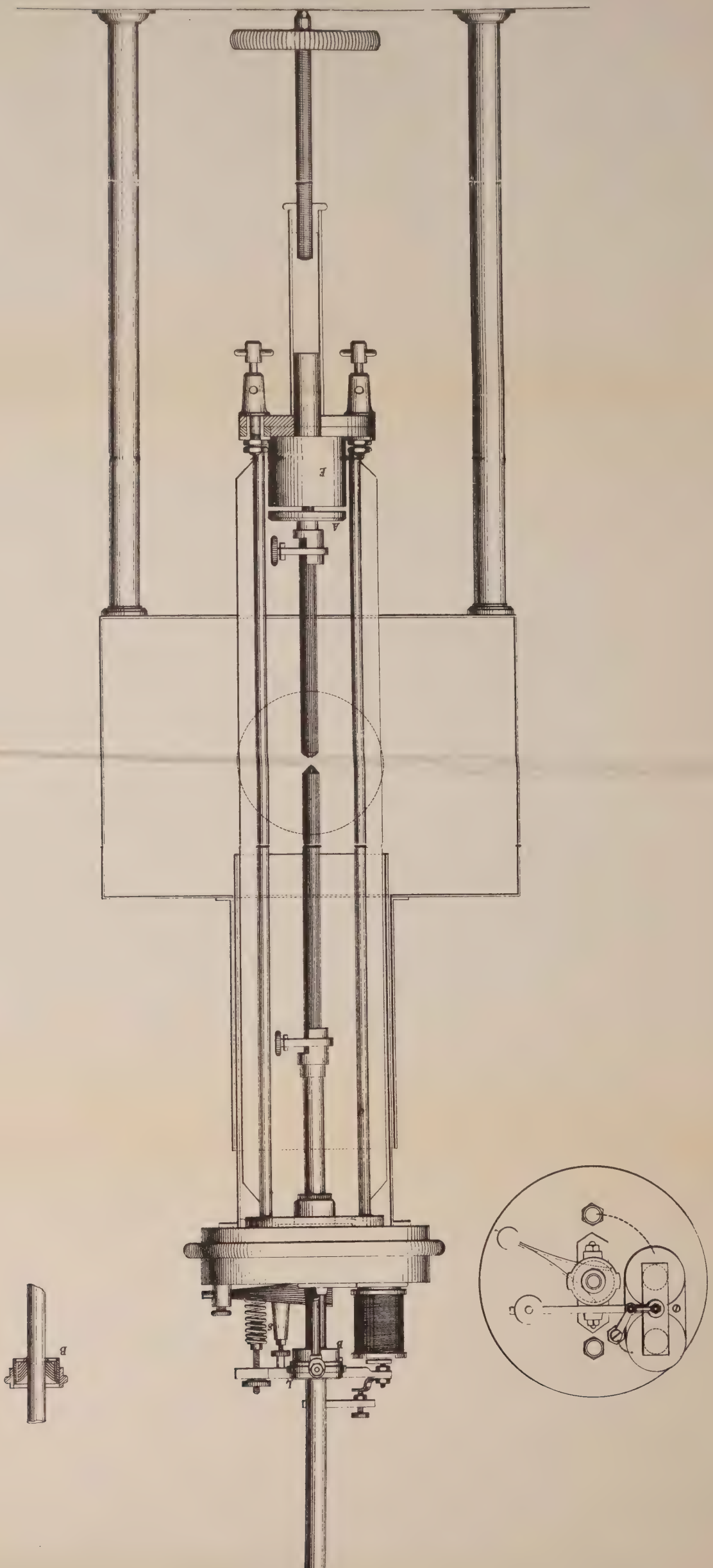
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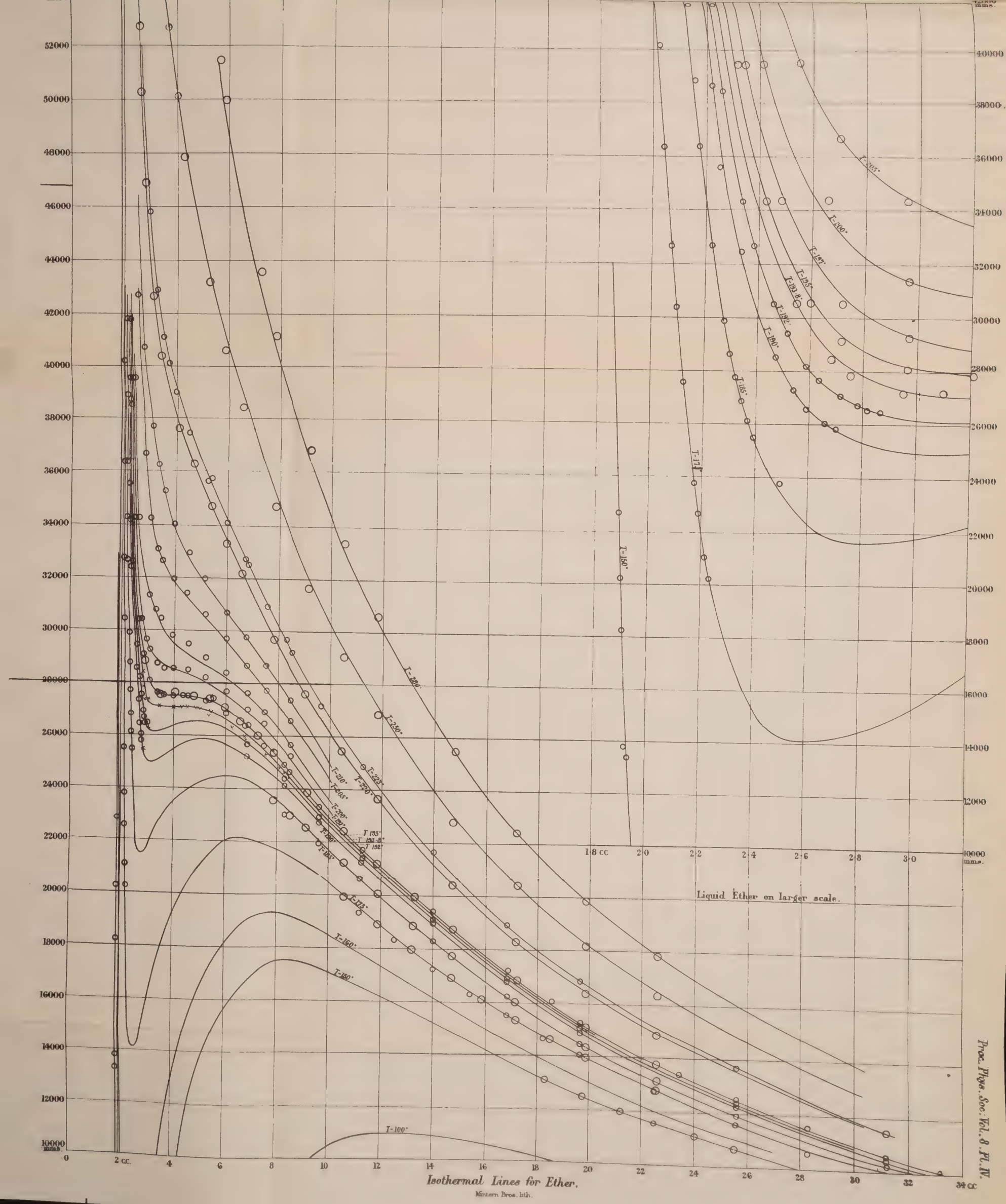

Isothermal Lines for Ether.

Mintern Bros. lith.

Proc. Phys. Soc. Vol. 8. Pl. IV.

Isochoric Lines for Ether.

Mercury Press. Inch.

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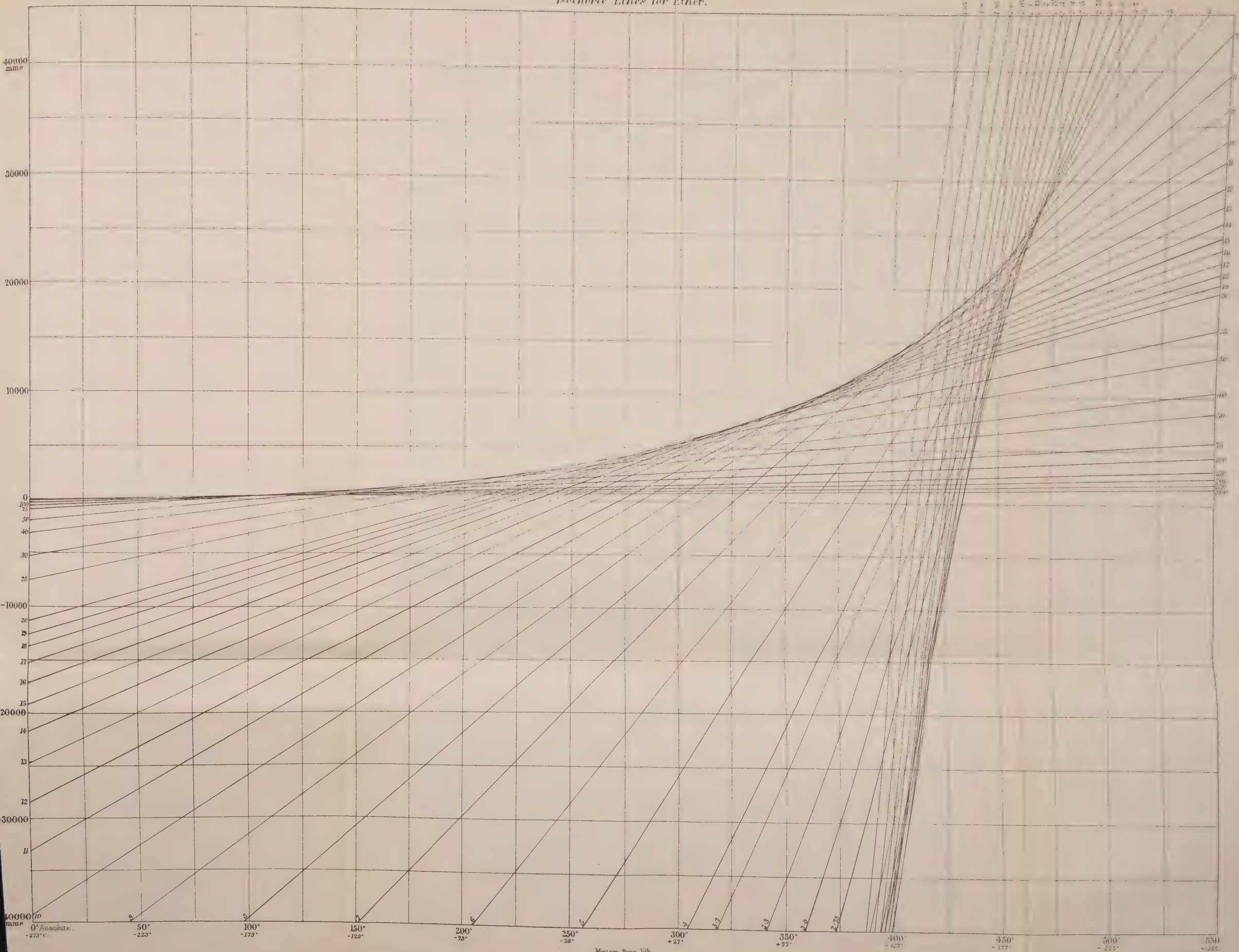
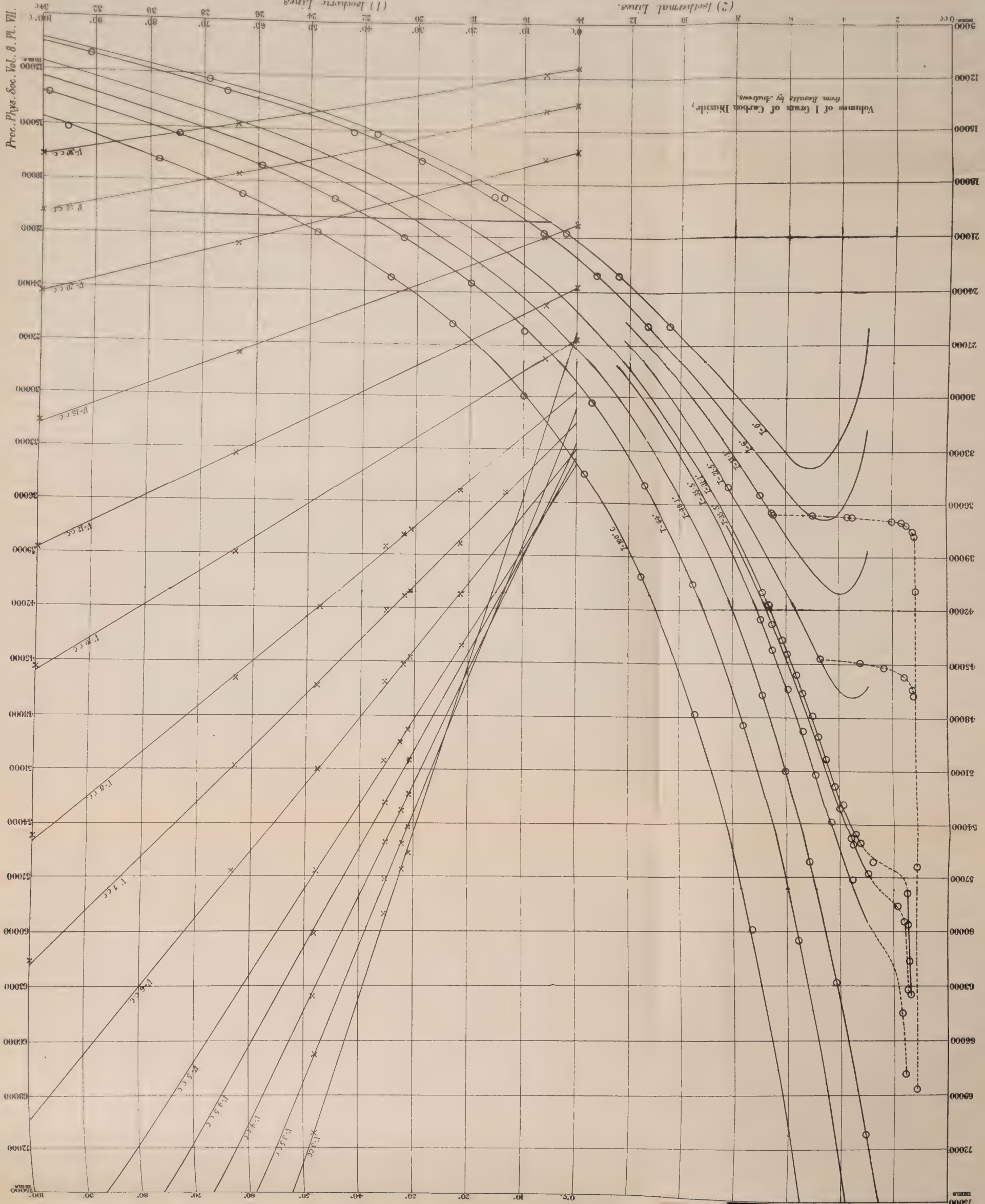
Isochoric Lines for Ether.




(1) Isochoric Lines.
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Volumes of 1 Gram of Carbon Dioxide, from Results by Andrews.
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PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY

OF LONDON.

SESSION 1886-87.

February 13th, 1886.

Dr. F. Guthrie, Vice-President, in the Chair.

The following were elected Members of the Society:—

Prof. T. H. Huxley, F.R.S.; Mr. A. E. Mills.

The following communications were made:—

"On Experimental Errors in Calorimetrical Work, and on Delicate Calorimetrical Thermometers." By Prof. S. U. Pickering.

"On a new Calorimeter." By Prof. W. F. Barrett.

"On a form of Calorimeter." By Prof. S. P. Thompson.

February 27th, 1886.

Dr. Gladstone, Vice-President, in the Chair.

The following were elected Members of the Society:—

Mr. D. E. Jones; Dr. Sydney Young.

The following communications were made:—

"Notes on the Thermodynamic Relations in the Paper of Messrs. Ramsay and Young." By Prof. W. C. Unwin.

"On a Map of the World in which the proportion of areas is retained." By Mr. Walter Baily.

"On a Delicate Calorimetric Thermometer." By Prof. S. U. Pickering.

March 13th, 1886.

Prof. Balfour Stewart, President, in the Chair.

The following were elected Members of the Society:—

A. C. Curtis-Hayward, M.A.; Prof. S. U. Pickering.

The following communications were made:—

"On the Growth of Filiform Silver." By Dr. J. H. Gladstone.

"On Apparatus for measuring the Electrical Resistance of Liquids." By Prof. A. W. Reinold.

"On Chromatic Photometry." By Capt. Abney and Major-Gen. Festing.

March 27th, 1886.

Prof. W. G. Adams, Vice-President, in the Chair.

The following was elected a Member of the Society:—

Mr. A. R. Wright.

The following communications were made:—

"On an Arc Lamp convenient for use with the Duboscq Lantern for projection." By Prof. S. P. Thompson.

"On a modified Maxwell's Galvanometer suitable for Lecture-purposes." By Prof. S. P. Thompson.

"On the Expansion of Mercury from its Melting-point to Zero Centigrade." By Profs. Ayrton and Perry.

"On the Deformation of a Straight Bar of Brass produced by amalgamating one edge." By Profs. Ayrton and Perry.

April 10th, 1886.

Prof. Balfour Stewart, President, and afterwards Prof. W. G. Adams, Vice-President, in the Chair.

The following communications were made:—

"On the Cause of the Solar-Diurnal Variations of Terrestrial Magnetism." By Prof. Balfour Stewart.

"On a Relation between the Critical Temperatures of Bodies and their Thermal Expansions as Liquids." By Prof. A. W. Rücker and Prof. T. E. Thorpe.

May 8th, 1886.

Prof. H. M'Leod, Vice-President, in the Chair.

The following was elected a Member of the Society:—

W. A. Price, M.A.

The following communications were made:—

"On a modified form of Wheatstone's Rheostat." By Mr. Shelford Bidwell.

"On a Theorem relating to Curved Diffraction-Gratings." By Mr. Walter Baily.

"On some Thermodynamical Relations."—Part IV. By Prof. W. Ramsay and Dr. S. Young.

May 22nd, 1886.

Prof. Balfour Stewart, President, in the Chair.

The following were elected Members of the Society:—

Mr. Chichester A. Bell, M.B.; Mr. W. Claude Johnson, F.R.A.S.; Mr. James Swinburne.

The following communications were made:—

"On the Sympathetic Vibrations of Jets." By Mr. Chichester A. Bell.

"On some Thermodynamical Relations."—Part V. By Prof. W. Ramsay and Dr. S. Young.

Exhibition of the Effects of Explosives by Dr. H. Sprengel.

Explanation by Mr. C. V. Boys of Thatcher's Slide-Rule, exhibited by Mr. Stanley.

June 12th, 1886.

Dr. J. H. Gladstone, Vice-President, in the Chair.

The following were elected Members of the Society :—

Mr. S. Rideal, B.Sc. ; Mr. E. C. Rimington.

The following communications were made :—

"On an Electric-light Fire-damp Indicator." By Walter Emmott and Wm. Ackroyd.

"Sur la distinction entre les raies d'origine solaire et d'origine terrestre." Par Dr. le Prof. Cornu.

"On a Hyperbolagraph." By Mr. H. H. Cunynghame.

"On a Voltaic Cell with Solid Electrolytes." By Mr. Shelford Bidwell.

June 26th, 1886.

Prof. W. E. Ayrton, Vice-President, in the Chair.

The following was elected a Member of the Society :—

Mr. E. M. Langley.

The following communications were made :—

"On certain Sources of Error in connexion with Experiments on Torsional Vibrations." By Mr. Herbert Tomlinson.

"On a Mode of maintaining Tuning-Forks by Electricity." By Prof. S. P. Thompson.

"Further Notes on the Formulæ of the Electromagnet and of the Dynamo." By Prof. S. P. Thompson.

November 13th, 1886.

Prof. Balfour Stewart, President, in the Chair.

The death of Dr. Guthrie, which occurred on the 21st October, was announced.

The following communications were made:—

"On the peculiar Sunrise-Shadows of Adam's Peak in Ceylon." By the Hon. R. ABERCROMBY.

"Note on the Internal Capacity of a Thermometer." By Mr. A. W. CLAYDEN.

November 27th, 1886.

Prof. W. G. ADAMS, Vice-President, in the Chair.

The following communications were made:—

"Note on the Measurement of the Coefficient of Mutual Induction of Two Coils." By Prof. G. C. FOSTER.

"On the Stability of Liquid Surfaces of Revolution." By Prof. A. W. RÜCKER.

December 11th, 1886.

Prof. H. MCLEOD, Vice-President, in the Chair.

The following were elected Members of the Society:—

Mr. W. L. NATANSON; Mr. E. NATANSON; Hon. R. ABERCROMBY; Mr. H. M. ELDER, M.A.; Mr. JULIUS WERTHEIMER, B.Sc.

The following communications were made:—

"On the Influence of Change of State from Liquid to Solid on Vapour-Pressure." By Prof. W. RAMSAY and Dr. S. YOUNG.

"On the Nature of Liquids, as shown by a study of the Thermal Properties of Stable and Dissociable Compounds." By Prof. W. RAMSAY and Dr. S. YOUNG.

"On Cauchy's Theory of Reflection and Refraction of Light." By Mr. JAMES WALKER.

"On a Voltaic Cell in which the Electrolyte is a Dry Metallic Oxide." By Mr. SHELFORD BIDWELL.

January 22nd, 1887.

Prof. H. MCLEOD, Vice-President, in the Chair.

The following was elected a Member of the Society:—

Mr. FREDERICK WOMACK, B.Sc., M.B.

The following communications were made :—

"On the Temporary and Permanent Effects on Iron of Raising its Temperature to 100° C." By Mr. H. TOMLINSON.

"On some new Measuring-Instruments used in Testing Materials." By Prof. W. C. UNWIN.

Mr. BOYS exhibited Spectrum Photographs made with Rowland's Diffraction-Grating.

The Members adjourned to Prof. UNWIN's Laboratory to inspect the new Measuring-Instruments.

Annual General Meeting.

February 12th, 1887.

Prof. BALFOUR STEWART, President, in the Chair.

The following Report of the Council was read by the President :—

In presenting their Report for the past Session, the Council have for the first time to record the loss of one of those who have taken a prominent part in the work of this Society. The death of Dr. Guthrie has been felt as a personal sorrow by all those who have attended our meetings, and especially by the members of the Council, with whom his constant presence at their meetings brought him into intimate relationship. The Society owes to Dr. Guthrie not only its foundation but that ever-ready and enthusiastic support which his genial character and scientific attainments, as well as his position as Professor of Physics in this place, rendered of the highest value.

A Committee having been formed, under the presidency of Prof. Huxley, for the purpose of raising a Memorial Fund in recognition of Dr. Guthrie's scientific labours, to be devoted to placing his children as nearly as might be possible in the position which they would have occupied but for his untimely death, and to supplementing the provision for his widow, the Council of this Society, feeling that no more suitable tribute could be offered to his memory, issued an appeal to our Members urging them to give their support to this fund; and they have been gratified to learn that this appeal has met with a liberal response.

The Council have heard with regret from Dr. Atkinson, who has

filled the office of Treasurer from the foundation of the Society, that he finds himself no longer able to attend to the duties of that office. The Society owes the satisfactory state of its finances to Dr. Atkinson's watchful care, and the Council desire to express their high appreciation of his services. Members will have noticed with pleasure that Professor Rücker, who has succeeded Dr. Guthrie as Professor of Physics at the Normal School of Science, South Kensington, has consented to be nominated as a candidate for the vacant office of Treasurer. Without dwelling on Professor Rücker's high scientific reputation, the Council desire to direct attention to the fact that his election to an important office in the Society would maintain the close connexion which has existed since its foundation between this Society and the Chair of Physics, from which connexion the Society has derived very great advantages.

There has been some diminution during the past Session in the number and extent of the papers contributed to the Meetings. For example, the April, July, and November parts of our 'Proceedings' in 1885 comprised altogether 33 papers, occupying 306 pages; while the corresponding parts in 1886 comprised only 25 papers, occupying 169 pages. The Council would point out to Members the desirability of bringing forward their original work in Physics before our Meetings. A contribution to our 'Proceedings' obtains immediate publication in the columns of the 'Philosophical Magazine,' and 100 copies of the paper are sent to the author, and the papers are once a quarter reprinted and sent to all Members of our Society, so that such papers obtain a wide circulation at an early date and in a convenient form. Moreover, the use of the magnificent collection of apparatus, under the control of the Professor of Physics, is freely extended to the authors of papers for the purpose of performing experiments at our meetings, so that the experimental illustration of physical papers can be carried out at our meetings with a completeness which is elsewhere impossible.

Among the papers read during the past Session the Council desire to call particular attention to a paper by Prof. Cornu on "Solar and Terrestrial Lines in the Spectrum," one by Dr. Balfour Stewart on the "Causes of Changes in Terrestrial Magnetism," and one by Prof. Rücker on the "Stability of Liquid Films."

The Library of the Society appears to be hardly as well known to Members as it deserves; it consists of a considerable number of volumes, having been much increased in the last few years by presents from Dr. Guthrie and Lady Siemens. The Library is kept

in a room adjoining the lecture-room in which our meetings are held, and books may be taken out by Members on application to the Librarian, Mr. C. V. Boys.

The second volume of Dr. Joule's works has been issued to Members, so that this undertaking is now completed.

The Council hope that the publication of Volta's works may be carried forward during the ensuing year.

The following additions have been made to the Library during the past Session:—

Proceedings of the Academy of Natural Sciences, Philadelphia.
Proceedings of the Institute of Mechanical Engineers.
Proceedings of the Royal Society.
Proceedings of the Cambridge Philosophical Society.
Proceedings of the Canadian Institute.
Scientific Proceedings of the Royal Dublin Society.
Journal of the Physico-Chemical Society of St. Petersburg.
Journal de Physique.
Journal of the Society of Arts.
Journal of the Society of Telegraphic Engineers.
Electrical Review.
Annales des Physik und Chemie.
Beiblätter des Physik und Chemie.
Séances de la Société Français de Physique.
Den Norske Nordhavs-Expedition. Zoologi: Mollusca and Crustacea.
Tokio Memoirs.
Medical and Biological Catalogue, University College, London.
Clarke's Transit Tables.

Application having been made from time to time for election to Membership of this Society by persons residing abroad, who have not been able to find any proposers personally acquainted with them, it has seemed desirable to the Council that a modification of the Bye-laws should be made so as to enable such persons to be eligible if Members of some recognized Scientific Society; and the Council have accordingly decided to propose the following addition to section 11 of the Bye-laws (under which Section personal knowledge of a Candidate is required), viz.:—

> "Provided that when a Candidate living abroad is a Member of a recognized Scientific Society such Membership may (subject to the approval of the Council) be held equivalent to the personal knowledge aforesaid."

The Council are indebted to Prof. Carey Foster for the following memoir.

By the death of Professor Frederick Guthrie all who have been in the habit of attending the meetings of the Physical Society will feel that they have lost a personal friend. His unfailing kindliness and geniality and his constant readiness to be helpful, without ever seeming conscious that any one owed him thanks, produced towards him a feeling of warm regard on the part of all who were in any way brought into frequent personal contact with him. Towards the Members of this Society he stood in a peculiar relation. It is seldom that a scientific Society owes its existence so distinctly to the action of one man as the Physical Society of London owes its origin to the action of Frederick Guthrie. In the summer of 1873, Guthrie invited his scientific friends and all those who, from their known tastes or official position, he thought likely to take interest in the scheme, to take steps for the formation of a Society for the cultivation of the science of Physics. The circular that he sent out ran as follows:—

"PHYSICAL SOCIETY.

"I wish to try to form a Society for Physical Research: for showing new physical facts and new means for showing old ones: for making known new home and foreign physical discoveries, and for the better knowledge one of another of those given to physical work. You who care for the being of such a Society, and are willing to help in its making, are hereby asked to write to me to that purpose before the 1st of October next. Whereupon you will be asked to meet so as to talk over the means.

"Frederick Guthrie,
24 Stanley Crescent, Notting Hill, W."

The proposal thus quaintly made met with so much support that the Society was constituted, and held its first meeting for the reading of papers on the 21st of March, 1874. None who knew Guthrie could think that any desire for personal prominence prompted him to take the lead in establishing this Society. His account of the matter was that, by virtue of his official position he was one of those upon whom the duty of promoting the progress of Physics in London chiefly lay; and in his opinion a Society such as he proposed, and such as has come into existence, was likely to be one of the most effectual means of attaining this end. So far, the only Society in London which concerned itself directly with Physics was the Royal Society; and Guthrie thought that, without in any way interfering with the field of action of the Royal Society, there was

ample room for a Society which should take cognizance of smaller matters, points of technical detail, useful laboratory contrivances, experimental methods of illustrating physical principles, questions connected with methods of teaching, and other things of much import to the advance of our science, which would nevertheless be out of place before the Royal Society. Guthrie felt also that in a Society devoted specially to Physics, the actual exhibition of physical phenomena to the members might usefully be made an important part of the proceedings. The position which the Physical Society now occupies and the value of the work it has done form the best proof of the correctness of these views. Guthrie, however, did more than merely throw out a happy suggestion at a time when many others were ready to take it up and act upon it. Without the efficient and energetic aid that he gave towards the practical realization of his idea, it is safe to say that the Physical Society would not yet have attained to its actual position of usefulness and influence among the Scientific Societies of London. Through his official connexion with the Science and Art Department, he was able to obtain from the Lords of the Committee of Council on Education permission for the Society's meetings to be held in the Physical Lecture-room of the Science Schools of the Department at South Kensington, and for the resources of the Physical Laboratory to be used at his discretion for the experimental illustration of subjects brought before the Society. More than this, he gave his time and personal efforts without stint to promoting the convenience of the authors of communications brought before the Society, and to increasing in all possible ways the interest and advantage of the Meetings.

In this room and at the first Anniversary Meeting of the Society since Dr. Guthrie's death, it is impossible, when recalling the loss we have sustained, not to remember first the genial friend who is no longer with us, and next to call to mind the almost parental relation in which he stood to the Society as a whole.

Frederick Guthrie was born in Bayswater in October 1833, and was the youngest of six children. One of his brothers, Mr. Francis Guthrie, is a distinguished mathematician, and has been for many years Principal of the South-African College, Cape Town. Guthrie received the greater part of his education at University College School and University College, London, where most of his time in the latter part of his course was given to Chemistry and Mathematics. He graduated as Bachelor of Arts of the University of

London in 1852. In the following spring he went to Heidelberg to work at Chemistry under Bunsen, and afterwards removed to Marburg in order to make a special study of Organic Chemistry under Kolbe. Here he took the degree of *Philosophiæ Doctor* in 1855. On his return to England he was appointed, in 1856, Assistant to Dr. Frankland, who was Professor of Chemistry in the then recently established Owens College, Manchester. In 1859 he became Assistant to the Professor of Chemistry in the University of Edinburgh, now Sir Lyon Playfair. After holding this appointment for about two years he went to Mauritius (in 1861) as Professor of Chemistry and Physics in the Royal College, and remained there till 1867, when he returned to England on leave. One of Guthrie's colleagues in Mauritius was Mr. Walter Besant, the eminent novelist, with whom he formed a life-long friendship. In a lecture given before the Society of Arts last February, Guthrie thus alludes to the duties which he and his friend had to discharge:—"We teachers," he says, "were expected to make ourselves generally useful. Thus, while I was supposed to teach Chemistry, Physics, and the Natural Sciences, my more gifted friend taught, amongst other things, Latin, Greek, Mathematics, and English Literature." But, however miscellaneous his official duties, Guthrie found time during this period for a large amount of valuable scientific work. In was in Mauritius that he carried out his first published physical investigations, which are recorded in three papers communicated to the Royal Society in 1864 and 1865, namely two papers "On Drops" and one "On Bubbles." These are excellent examples of purely experimental investigations, well-planned and carefully carried out. No doubt it was in his work as a practical chemist that the properties of Drops and Bubbles first attracted his attention; but it is characteristic of much of his subsequent work, and a striking evidence that he possessed the true spirit of the scientific investigator, that he saw matter for careful study and experiment in the most familiar things, instead of, like most of us, letting familiarity with the outward look of things blind him to how little accurate knowledge of them we possess.

In 1869 Guthrie was appointed Lecturer on Physics in the Royal School of Mines, and retained this post (with altered title and widened duties) till his death, which occurred, almost suddenly, from cancer of the throat, in October 1886, when he had just completed his 53rd year.

He was elected a Fellow of the Royal Society of Edinburgh in

1859 and a Fellow of the Royal Society of London in 1873. He published some forty papers on Chemistry and Physics, the Chemical papers, about one third of the whole number, being chiefly published in the first eight or nine years after his *promoviring* as Doctor. Those which followed the investigation into the properties of Drops were almost wholly physical, though many of them,—as, for example, the important series of papers entitled "On Salt-Solutions and Attached Water,"—dealt with matters which had a chemical as well as a physical aspect. Many of his principal investigations were communicated in the first instance to this Society, and may be supposed to be familiar to many of the Members. Prior to the formation of this Society, however, he had made the remarkable discovery which he described as "Approach caused by Vibration," as seen, for example, in the apparent attraction exerted by a vibrating tuning-fork on a light object suspended in the air near it. These phenomena, which were discovered independently and about the same time by Sondhaus, are closely allied to those which have been studied so elaborately, both mathematically and experimentally, in the case of incompressible fluids, by Professor Bjerknes, of Christiania, and have been followed up, in the case of air, in the beautiful experiments of Mr. Stroh.

But fertile and industrious as Guthrie was as an experimentalist, it is perhaps even more as a teacher than as an investigator that his influence will be felt hereafter. The course of work which he devised for the "Certificated Science Teachers" who came under his instruction at South Kensington was an entirely new departure in the teaching of Physics. Physical laboratories for the instruction of students had already begun to appear, but in them men were taught for the most part to use ready-made instruments. Guthrie's plan was to teach his pupils not only how to use, but how to make their instruments for themselves. Considering how absolutely the value of the system of instruction in science that has been organized under the Science and Art Department depends upon the pupils being made familiar with actual facts and phenomena, and not merely with descriptions or statements about them, it may be regarded as a matter of national importance that for the last fourteen or fifteen years every Science Teacher, who has gone through the course of Physics at South Kensington, has gone back to his classes not only with a stock of thoroughly serviceable instruments, but that he has learned how to make them for himself at a merely nominal cost, and can, if he chooses, spread the art among his pupils, and thus make

his teaching of a thoroughly genuine and practical kind. How earnestly convinced Guthrie himself was of the value of this plan may be seen from his lectures on "Science Teaching," delivered before the Society of Arts last spring.

It is difficult for those who knew Guthrie well not to think of him much more as a man and a friend than as a man of science, Though he was one of the most independent and self-reliant of men both in thought and in action, in everything that concerned others he carried charity in judgment and generosity, indeed, almost to a fault. This is not the place in which to speak of his private life and affairs; but it may be said that he had to encounter difficulties such as few men have to meet, and that he faced them with such calmness and courage that most even of his intimate friends knew nothing of their existence until after his death.

In Frederick Guthrie we have lost a strong, a true, and a good man.

The Treasurer made the Financial Statement given below (pp. 15 and 16).

The Society then proceeded to the election of Officers and other Members of Council for the ensuing year. The Council was constituted as follows:—

President.—Prof. Balfour Stewart, LL.D., F.R.S.

Permanent Vice-Presidents.—Dr. J. H. Gladstone, F.R.S.; Prof. G. C. Foster, F.R.S.; Prof. W. G. Adams, M.A., F.R.S.; Sir William Thomson, D.C.L., LL.D., F.R.S.; Prof. R. B. Clifton, F.R.S.

Other Vice-Presidents.—Dr. E. Atkinson; Prof. W. E. Ayrton, F.R.S.; Shelford Bidwell, M.A., LL.B., F.R.S.; Prof. H. McLeod, F.R.S.

Secretaries.—Prof. A. W. Reinold, M.A., F.R.S.; Walter Baily, M.A.

Treasurer.—Prof. A. W. Rücker, M.A., F.R.S.

Demonstrator and Librarian.—C. Vernon Boys.

Other Members of Council.—R. H. M. Bosanquet, M.A.; W. H. Coffin; Conrad W. Cooke; Prof. G. Forbes, F.R.S.E.; Prof. F. Fuller, M.A.; Prof. J. Perry, F.R.S.; W. N. Shaw, M.A.; Prof. S. P. Thompson, D.Sc.; G. M. Whipple, B.Sc.; C. R. Alder Wright, D.Sc., F.R.S.

Votes of thanks were passed to the Lords of the Committee of Council on Education; to the President and other Officers of the Society; and to the Auditors.

THE TREASURER IN ACCOUNT WITH THE PHYSICAL SOCIETY, FROM JANUARY 1ST, 1886, TO DECEMBER 31ST, 1886.

Dr.	£ s. d.	£ s. d.
Balance in Bank, January 1, 1886		116 18 6
Subscriptions for Exhibition		3 12 0
Entrance-Fees	19 0 0	
Subscription for 1880	1 0 0	
Subscriptions for 1881	2 0 0	
„ 1882	2 0 0	
„ 1883	4 0 0	
„ 1884	7 0 0	
„ 1885	19 0 0	
„ 1886	104 0 0	
„ 1887	7 0 0	
17 Life Compositions	170 0 0	
		335 0 0
One year's Dividend on £400 Furness 4 per cent. Debenture Stock, less Income Tax	15 19 4	
One year's Dividend on £575 Midland 4 per cent. Preference Stock, less Income Tax	22 4 8	
One year's Dividend on £200 Lancaster Corporation Stock, less Income Tax	7 14 8	
One year's Dividend on £200 Metropolitan Board of Works Stock, less Income Tax	6 15 4	
		52 4 0
Balance due to Treasurer		3 7 8
		£511 2 2

Cr.	£ s. d.	£ s. d.
Due to Treasurer		5 10 4
Taylor and Francis :—		
Proceedings, vol. vii. (parts 3 and 4)	59 17 6	
Postage and addressing	7 1 0	
Members' separate copies	11 16 6	
Miscellaneous printing	12 15 10	
Joule, vol. ii.	40 0 0	
		131 10 10
Reporting to July		10 10 0
Attendance at Meetings and incidental expenses		7 9 8
Petty Cash :—		
Mr. Reinold	2 4 6	
Mr. Baily	1 0 0	
Mr. Atkinson	1 2 0	
		3 6 6
Balance in Bank		352 14 10
		£511 2 2

Audited and found correct,

London, February 12th, 1887.

E. ROBERT FESTING,
GEORGE FULLER, } *Auditors.*

PROPERTY ACCOUNT OF THE PHYSICAL SOCIETY, DECEMBER 31, 1886.

ASSETS.	£	s.	d.
Balance in Bank	352	14	10
Subscriptions due	100	0	0
£400 Furness 4 per cent. Debenture Stock	456	0	0
£575 Midland 4 per cent. Preference Stock	650	0	0
£200 Metropolitan Board of Works Stock	217	0	0
£200 Lancaster Corporation Stock	224	0	0
Sales for 1886, per Taylor and Francis	14	0	0
	£2013	14	10

LIABILITIES.	£	s.	d.
Due to Treasurer	3	7	8
Subscriptions for 1887	7	0	0
Bills outstanding, Williams and Norgate	4	16	0
Index to Joule	15	0	0
Taylor and Francis, Two Parts Proceedings &c.	57	4	3
Due on second volume of Joule	132	13	9
Balance	1793	13	2
	£2013	14	10

We have examined the above Account, and also the Securities at the Bank, and find the same to be correct.

London, February 12th, 1887.

E. ROBERT FESTING,
GEORGE FULLER, } *Auditors.*